Isospin Observables near the Fermi Energy
(in the spirit of a workshop)

Isospin equilibration - N/Z mixing
Flow
Isotopic composition of fragments
Particle spectra

N/Z Equilibration
N/Z equilibration is one way to get a handle on the symmetry energy of the nuclear EOS

PLB 1994

53 MeV/nuc
E/A=35.45 MeV
$^{40}$Ca, $^{40}$Ar + $^{58}$Fe, $^{58}$Ni
$\Theta_{\text{lab}} = 40^\circ$

PLB 1996

projectile neutrons       target neutrons
projectile protons        target protons

Li, PRC 1995
iBUU shows clear translucency at 45 Mev/nucleon and above

\[ \text{Isotope tracer as study of nuclear stopping} \]

\[ R_z = \frac{2(Z_{\text{det}}^0 - Z_{\text{det}}^{Zr} - Z_{\text{det}}^{Ru})}{Z_{\text{det}}^{Zr} - Z_{\text{det}}^{Ru}} \]

400 MeV/nucleon

FOPI: Rami, PRL2000
Deep Inelastic Transfer mechanism can be used to study N/Z equilibration


Isospin Diffusion

\[ R_i = \frac{2\delta - \delta_{PP} - \delta_{TT}}{\delta_{PP} - \delta_{TT}} \]

- symmetry energy will act as a driving force to transport the n or p between projectile to target

Difference between projectile and target spectator asymmetry, 
\[ \delta = \frac{(N-Z)}{(N+Z)} \], measures the isospin diffusion which can be used to extract information about symmetry energy.

\[ {}^{112,124}_{\text{Sn}}, \ 50 \text{ MeV/nucleon} \]

Tsang et al PLB 2004 and others
• Central collisions – learn about nuclear translucency
• Peripheral collisions – learn about transport
• Both should depend on EOS

Isobaric and isotopic projectile fragmentation with FAUST

• N/Z Equilibration
• Determining the N/Z of the Quasiprojectile Source
  – Reconstruction
  – Yield ratio method

<table>
<thead>
<tr>
<th>System</th>
<th>Tgt N/Z</th>
<th>Proj N/Z</th>
<th>ΔN/Z</th>
<th>N/Z_CS</th>
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<tbody>
<tr>
<td>40Ar+112Sn</td>
<td>1.24</td>
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<td>0.02</td>
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<td>40Ar+124Sn</td>
<td>1.48</td>
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<td>0.26</td>
<td>1.41</td>
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<td>40Ca+112Sn</td>
<td>1.24</td>
<td>1.00</td>
<td>0.24</td>
<td>1/17</td>
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<tr>
<td>40Ca+124Sn</td>
<td>1.48</td>
<td>1.00</td>
<td>0.48</td>
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<tr>
<td>48Ca+112Sn</td>
<td>1.24</td>
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<td>48Ca+124Sn</td>
<td>1.48</td>
<td>1.40</td>
<td>0.08</td>
<td>1.46</td>
</tr>
</tbody>
</table>

32 and 45 MeV/nucleon

\[(N/Z)_{\text{Composite System}} = \frac{N_{\text{Projectile}} + N_{\text{Target}}}{Z_{\text{Projectile}} + Z_{\text{Target}}}\]
Reconstruction

- Quasiprojectile $Z$  
  - Know fragment $Z$  
  \[ Z_{qp} = \sum Z_f \]

- Apparent Quasiprojectile $A$  
  - Know fragment $A$  
  \[ A_{qp} = \sum A_f \]

- Apparent Quasiprojectile Excitation Energy  
  - Using the balance of energy  
  \[ E^*_qp = \sum (m_f + E_{f_{cms}}) - m_{qp} \]

- Cut on $Z_{qp} = Z_{beam}$

---

Reconstruction Results

<table>
<thead>
<tr>
<th>System</th>
<th>32 MeV/u</th>
<th>45 MeV/u</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{40}$Ar + $^{115}$Sn</td>
<td>1.24</td>
<td>1.24</td>
</tr>
<tr>
<td>$^{40}$Ar + $^{124}$Sn</td>
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<td>1.48</td>
</tr>
<tr>
<td>$^{48}$Ca + $^{112}$Sn</td>
<td>1.24</td>
<td>1.24</td>
</tr>
<tr>
<td>$^{48}$Ca + $^{124}$Sn</td>
<td>1.48</td>
<td>1.48</td>
</tr>
<tr>
<td>$^{48}$Ca + $^{125}$Sn</td>
<td>1.48</td>
<td>1.48</td>
</tr>
</tbody>
</table>

Undetected neutrons cause reconstruction to under predict the N/Z

Fragmentation effected by the N/Z of the quasiprojectile source

If fragment identity is set early fragments can be used to determine N/Z of quasiprojectile
Global Fitting of Yield Ratios

- Simple equation stating that the N/Z of the quasiprojectile source is some part target and some part projectile

\[
\left( \frac{N}{Z} \right)_S = X \left( \frac{N_T}{Z_T} \right) + Y \left( \frac{N_P}{Z_P} \right)
\]

Yield Ratio Method Results

<table>
<thead>
<tr>
<th>Ratio</th>
<th>N/Z Projectile</th>
<th>N/Z Target</th>
<th>N/Z Source</th>
<th>Target (X)</th>
<th>Projectile (Y)</th>
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<tbody>
<tr>
<td>^3He</td>
<td>0.6579</td>
<td>1.23</td>
<td>1.24</td>
<td>1.227731</td>
<td>0.506890</td>
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<tr>
<td>^7Li/^7Be</td>
<td>0.1601</td>
<td>1.40</td>
<td>1.48</td>
<td>1.320859</td>
<td></td>
</tr>
<tr>
<td>^6He/Li</td>
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<tr>
<td>^6He/Li</td>
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<td>1.40</td>
<td>1.48</td>
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<td>m</td>
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<tr>
<td>^6He/Li</td>
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<td>1.40</td>
<td>1.48</td>
<td>1.430993</td>
<td>b</td>
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<tr>
<td>^6He/Li</td>
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<td>1.22</td>
<td>1.24</td>
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Yield Ratio Method Results

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<tr>
<th>System</th>
<th>N/Z</th>
<th>N/Z Exp.</th>
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<tr>
<td>^6Ar + ^115Sn</td>
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<td>1.24</td>
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<tr>
<td>^40Ar + ^124Sn</td>
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<td>1.48</td>
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<tr>
<td>^40Ca + ^115Sn</td>
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<td>1.17</td>
</tr>
<tr>
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<td>^40Ca + ^115Sn</td>
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<td>1.29</td>
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<tr>
<td>^40Ca + ^115Sn</td>
<td>1.24</td>
<td>1.46</td>
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<td>1.29</td>
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<tr>
<td>^40Ca + ^115Sn</td>
<td>1.24</td>
<td>1.46</td>
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</tbody>
</table>
Simulation Results

TABLE XX. Summary of results from DIT/SMM comparison with experiment. 32 MeV/nucleon systems top half, 45 MeV/nucleon systems bottom half.

<table>
<thead>
<tr>
<th>System</th>
<th>N/Z</th>
<th>N/Z</th>
<th>Comp.</th>
<th>Exp.</th>
<th>Exp.</th>
<th>Sim.</th>
<th>Sim.</th>
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<td>0.99</td>
<td>1.23</td>
<td>1.23</td>
<td>1.17</td>
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<td>4.08</td>
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<td>$^{40}$Ar + $^{124}$Sn</td>
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<td>1.12</td>
<td>1.43</td>
<td>1.38</td>
<td>6.20</td>
</tr>
</tbody>
</table>

A simple way of finding the average amount of N/Z mixing

\[
\text{Percent Mixing} = \frac{N_{\text{source}}}{N_{\text{composite}}} \times 100
\]

\[
\text{N/Z}_{\text{cs}} = \frac{N_{\text{source}}}{N_{\text{composite}}}
\]

\[
\text{Observing 54%}
\]
Comparison with iBUU (Li & Chen)

48Ca+112Sn N/Z summary

Equilibration from Scaling of Yield Ratios of heavy residues

\[ R_{21}(N,Z) = \frac{Y_2}{Y_1} \]

\[ R_{21} = C \exp (\alpha N) \]

- \( {}^{86}\text{Kr}+^{124}\text{Sn}, {}^{112}\text{Sn} \)
  - (data inside \( \theta_{gr} = 6.2^\circ \))

Souliotis et al
Comments

- Globally, detected fragments reflect the N/Z at the time of fragment formation.
- Fragment ratios or fractional yields can be used to determine the N/Z of the source from which they originate – even if it is not totally reconstructed.
- Deep Inelastic collisions of $^{40,48}$Ca, $^{40}$Ar with $^{112,124}$Sn at 35 Mev/nucleon and $^{86}$Kr with $^{112,124}$Sn at 25 MeV/nucleon show partial but not complete N/Z mixing during the interaction.
- Current efforts entail trying to see what symmetry energy is required in dynamical codes to reproduced this level of N/Z mixing.

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Transverse Flow

Directed/Transverse/Sideward Flow

Examination of space-momentum correlation of particle emission in the reaction plane.

Quantifying Transverse Flow

Flow Parameter:

$$F(\text{MeV}/c) = \frac{\partial \langle P_x \rangle}{\partial Y_{\text{red}}} \bigg|_{Y=\text{mid-rapidity}}$$

Flow Sensitivity to $E_{\text{sym}}(\rho)$:
Transverse Flow

Neutron and Proton Transverse Flow
- Requires accurate energy and angular detection of neutrons

$^3\text{H}$ and $^3\text{He}$ Transverse Flow
- 20% larger flow for $^3\text{He}$ than $^3\text{H}$ for Asy-Stiff $E_{\text{sym}}$.

Flow Sensitivity to $E_{\text{sym}}(\rho)$:
Experimental Evidence from Direct Flow

INDRA/ALADiN Collaborations
• $t^3\text{He}$ and $^6\text{He}/^6\text{Li}$ differences are not clear.

Energy Dependence:
$^3$H vs. $^3$He Flow Parameter

- 20% larger flow predicted for $^3$He vs. $^3$H for “stiff” $E_{sym}$ for 55 MeV/u Fe+Fe.
- INDRA/ALADiN didn’t clearly see the shift for 100 MeV/u Xe + Sn systems.
- See a larger isospin sensitivity in the 55 MeV/u systems (see $Z=1$).

Would we see symmetry energy dependence for $^3$He and $^3$H flow in the lower energy range?


Experimental Results
$Z=1$ & $Z=2$ Fragments

Increased flow observed in more neutron rich system.

Results are in agreement with previous work of Pak. et al.
Pak. PRL. 78, 1022 (1997).

Note: $ZV_{||}$ Cut = 55%
Transverse Flow for Isotopic Fragments

Experimental Results
Isotopic Fragments

Increased flow is observed for $^3$He particles in comparison to $^3$H particles.

Increased flow for more neutron rich system.

Increased flow for 3He particles in comparison to 3H particles.

Linear Fit Range:
-0.4 to 0.4 \((Y/Y_{proj})_{cm}\).
It has been shown that the multiplicities and energy spectra of light clusters (such as $^2$H, $^3$H, $^3$He, $^4$He) are sensitive to the density dependence of the nuclear symmetry energy using iBUU/coalescence model.

Ratios corresponding to the A=3 mirror nuclei are very similar to those of the n/p.

Ratios of $^3$H-$^3$He are predicted to be sensitive to the density dependence of the nuclear asymmetry term.

A=3 mirror nuclei may provide a good measure of the density dependence of the asymmetry energy.

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**LCP Energy spectra as probe of Density Dependence of the Symmetry Energy**


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**Isobaric Yield Ratios from $^{58}\text{Fe, Ni} + ^{58}\text{Fe, Ni}$**

Hard: $x = -2$
Soft: $x = 1$

or of 2 neutrons

The $^3$H/$^3$He ratio for $^{58}\text{Fe} + ^{58}\text{Fe}$ compared to iBUU/coalescence calculations. $^{58}\text{Ni} + ^{58}\text{Ni}$ show similar trends.
Isobaric double yield ratio

- The $Y^3(H)/Y^3(He)$ ratio is roughly increased by a factor of 2 for the neutron rich system.
- Seen in the $^{112,124}Sn$ analysis from Famiano (private communication).

Energy spectra may not be best observable for lower energies at this time ...

Isoscaling studies as a function of $E^*/A_{QP}$ of the reconstructed QPs of $^{40}Ca + ^{112}Sn$ reaction at 45 MeV/A

$Z - V_{||}/c$ 2D distributions as a selector of QP

$$\alpha = \frac{4 C_{sym}}{\Delta}, \quad \Delta = \left[ \left( \frac{Z}{A} \right)^2 - \langle \left( \frac{Z}{A} \rangle \right)^2 \right]$$

$N/Z_{QP}$ zones for isoscaling studies as a function of $E^*/A_{QP}$
Isoscaling studies on the emitted fragments from the de-excitation of the reconstructed QPs on $^{40}$Ca $+ ^{112}$Sn system at 45 MeV/A

\[ E_b = 45 \text{ MeV/u} \]

$1.5 < \frac{E}{A_{QP}} < 2.5 \text{ MeV/u}$

$14 \ll Z_{QP} \ll 23$

\[ R = \frac{Y_2}{Y_1} \]

$E_b = 45 \text{ MeV/u}$

$2.5 < \frac{E}{A_{QP}} < 3.5 \text{ MeV/u}$

$14 \ll Z_{QP} \ll 23$

\[ R = \frac{Y_2}{Y_1} \]
$H_{\alpha} = 0.493 \pm 0.019$

$He_{\alpha} = 0.573 \pm 0.026$

$Li_{\alpha} = 0.637 \pm 0.029$

$Be_{\alpha} = 0.586 \pm 0.037$

$B_{\alpha} = 0.596$

$C_{\alpha} = 0.565 \pm 0.032$

$N_{\alpha} = 0.518 \pm 0.023$

$O_{\alpha} = 0.639 \pm 0.039$

$E_b = 45 \text{ MeV/u}$

$5.5 < E^*/A_{QP} < 6.5 \text{ MeV/u}$

$14 \leq Z_{QP} \leq 23$

$R = Y_2/Y_1$

$E_b = 45 \text{ MeV/u}$

$6.5 < E^*/A_{QP} < 7.5 \text{ MeV/u}$

$14 \leq Z_{QP} \leq 23$
$H_\alpha = 0.384 \pm 0.009$

$He_\alpha = 0.481 \pm 0.033$

$Li_\alpha = 0.526 \pm 0.009$

$Be_\alpha = 0.473 \pm 0.006$

$B_\alpha = 0.427$

$C_\alpha = 0.538 \pm 0.095$

$Eb = 45 \text{ MeV/u}$

$7.5 < E^*/A_{QP} < 8.5 \text{ MeV/u}$

$14 \leq Z_{QP} \leq 23$

$R = Y_2/Y_1$

$H \alpha = 0.337 \pm 0.003$

$He \alpha = 0.385 \pm 0.013$

$Li \alpha = 0.394 \pm 0.060$

$Be \alpha = 0.480 \pm 0.200$

$B \alpha = 0.439$

$Eb = 45 \text{ MeV/u}$

$8.5 < E^*/A_{QP} < 9.5 \text{ MeV/u}$

$14 \leq Z_{QP} \leq 23$
$^{40}$Ca + $^{112}$Sn @ 45 MeV/u

$14 \leq Z_{QP} \leq 23$

$\langle \alpha \rangle = 1.843 \pm 0.118$ $\langle E^*/A_{qp} \rangle = 2$ MeV/u

$\langle \alpha \rangle = 1.188 \pm 0.047$ $\langle E^*/A_{qp} \rangle = 3$ MeV/u

$\langle \alpha \rangle = 0.887 \pm 0.037$ $\langle E^*/A_{qp} \rangle = 4$ MeV/u

$\langle \alpha \rangle = 0.726 \pm 0.023$ $\langle E^*/A_{qp} \rangle = 5$ MeV/u

$\langle \alpha \rangle = 0.512 \pm 0.020$ $\langle E^*/A_{qp} \rangle = 6$ MeV/u

$\langle \alpha \rangle = 0.510 \pm 0.037$ $\langle E^*/A_{qp} \rangle = 7$ MeV/u

$\langle \alpha \rangle = 0.589 \pm 0.023$ $\langle E^*/A_{qp} \rangle = 8$ MeV/u
Neutron loss correction on the (N/Z)QP


\[40^{\text{Ca}} + 112^{\text{Sn}} @ 45 \text{ MeV/A}\]

DIT+SMM+FF

\[\text{DIT} = \text{DIT} + \text{SMM} + \text{FF} \]

(N/Z) real, weighted fit

\(\text{(N/Z)QP neutron loss corrected}\)

\(\text{N/ZQP}\)


D.V. Shetty et al., PRC 74 (2005) 024602

G.A. Souliotis et al., PRC 73 (2006) 024606

G.A. Souliotis et al., PRC 75 (2007) 011601

D.V. Shetty et al., PRC 76 (2007) 024606

D.V. Shetty et al., PRC 74 (2005) 024602

Decrease in Symmetry energy (Expt. Observation)

- G.A. Souliotis et al., PRC 73 (2006) 024606
- G.A. Souliotis et al., PRC 75 (2007) 011601
- A. Le Fevre et al., PRL 94 (2005) 162701
- D.V. Shetty et al., PRC 76 (2007) 024606
- D.V. Shetty et al., PRC 74 (2005) 024602
Decrease in Symmetry energy (Expt. Observation)

J. Iglio et al., PRC 74 (2006) 024605
C. Sfienti, NUFRA, Turkey (2007)

Decrease due to thermal expansion

Finite T Thomas-Fermi Seyler Blanchard interaction

S.K. Samaddar et al., PRC 76 (2007) 041602

Data: G.A. Souliotis et al., PRC 73 (2006) 024606
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Summary

- Symmetry energy appears to decrease with increasing excitation energy – this seems to be more a result of changes in density than temperature
- It is desirable to determine the density dependence of the symmetry energy – N/Z equilibration, Flow, fragment composition all appear to be promising observables
- Caveat: exact values currently extracted have some model dependence – but the overall trend seems robust
- Challenge to field is to get consistent description of all available data in a uniform framework

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A. Botvina, B.A. Li & L.W. Chen, M. Papa & A. Bonasara, M. Ditoro

Thanks to the Texas A&M University at Qatar for access to their computer cluster

This work was supported in part by:
The Robert A. Welch Foundation: Grant Number A-1266 and,
The Department of Energy: Grant Number DE-FG03-93ER40773