The LHCf data hadronic interactions and UHECR showers

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EXTREMELY ENERGETIC COSMIC-RAY EVENT*

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(Received April 12, 1961)

Problems of determination of:

Energy
Mass A

Hadronic interaction Modeling

~50 years of UHECR

Measure a single slice of the shower at the ground
EXTREMELY ENERGETIC COSMIC-RAY EVENT

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Hadronic interaction
Modeling

Different components

Measure a single slice of
the shower at the ground

~50 years of UHECR
The Fly's Eye Detector concept

"Quasi-Calorimetric" Energy Measurement

Fluorescence Light
\[ L(\Omega) \rightarrow F_\gamma(X) \rightarrow N_{e^\pm}(X) \]
\[ E \propto \int (\frac{dE}{dX}) \, dX \]

\[ E_{\text{ionization}} = \int dX \, N_e(X) \left\langle -\frac{dE}{dX} \right\rangle \]

\[ E_{\text{tot}} = E_{\text{ionization}} + E_\nu + E_\mu + E_{\text{ground}} \]
Area $\propto$ Energy

Shape depends on:
- Primary Identity
- Interaction Model

$E \propto \int \left(\frac{dE}{dX}\right) dX$
\[ E \approx 10^{20} \text{ eV} \]
Longitudinal Development
Shape studies

\[ X_{\text{max}} \]
“Fixed Target” measurements

Nucleus targets

Pion/kaon projectiles.

Cover entire
kinematical Phase Space
[including fast particles
in forward region]
COMPOSITION of UHECR

Very high astrophysical importance

Controversial - inconsistent observations.

$X_{\text{max}}$

Fluctuations of $X_{\text{max}}$

Other methods
Shower fluctuations

![Graph showing RMS($X_{max}$) vs Energy]
HIRES 2009

Fluctuations on $X_{\text{max}}$
$X_{\text{max}}$ and the Composition of Cosmic Rays

$$\langle X_A(E) \rangle \simeq \left\langle X_p \left( \frac{E}{A} \right) \right\rangle$$

$$\langle X_p(E) \rangle \simeq X_0 + D_p \log_{10} E$$

$$\langle X_A \rangle \simeq \langle X_p \rangle - D_p \log_{10} A$$
\[
\langle X_A \rangle \simeq \langle X_p \rangle - D_p \log_{10} A
\]

\[
\langle X_{\text{He}} \rangle \simeq \langle X_p \rangle - 30 \text{ g cm}^{-2}
\]

\[
\langle X_{\text{O}} \rangle \simeq \langle X_p \rangle - 60 \text{ g cm}^{-2}
\]

\[
\langle X_{\text{Fe}} \rangle \simeq \langle X_p \rangle - 90 \text{ g cm}^{-2}
\]
Measurements of $\langle \log A \rangle$

$$\langle \ln A \rangle_E = \frac{\sum_A \phi_A(E) \ln A}{\sum_A \phi_A(E)}$$
Measurements of Composition evolution.

\[
\frac{d\langle \ln A \rangle_E}{d \ln E} = 1 - \frac{D_{\text{exp}}}{D_p}
\]
The theoretical curves: \( <X_{\text{max}}(E)> \) are determined by the parameters that describe hadronic interactions. (and by their energy dependence).

Interaction Lengths
Multiplicity
Inclusive Spectra

\[
\left| \langle X_p \rangle_{\text{Model 1}} - \langle X_p \rangle_{\text{Model 2}} \right| \lesssim 20 \text{ g cm}^{-2}
\]

\[
D_p = \frac{d\langle X_{\text{max}} \rangle}{d \log_{10} E} \simeq 45 - 55 \text{ g cm}^{-2}
\]

\( 10^{19} \text{ eV} \)
Importance of “CORNERS”

Abrupt change in the variation of the properties of hadronic interactions with energy

Abrupt change in the composition evolution.

Fig. 25.— Comparison of current HiRes stereo $<X_{\text{max}}>$ results with results from the HiRes-prototype/MIA hybrid (Abu-Zayyad et al. 2001) and previously published HiRes stereo results (Abbasi et al. 2005).
Electromagnetic Showers

versus

Hadronic Showers

Toy model discussion.
Electromagnetic Shower

\[ E_{e^+} = E_\gamma \, u \]

\[ E_\gamma = E_e \, v \]

\[ \psi(u) \]

Pair production

\[ \varphi(v) \]

Bremsstrahlung

Radiation Length
(Energy independent)

Vertices:
theoretically understood
(and scaling)
Electromagnetic Showers

\[ X_{\text{max}}(E) \approx \lambda_{\text{rad}} \ln \left( \frac{E}{\varepsilon} \right) \]

\[ N_{\text{max}}(E) \approx \frac{E}{\varepsilon} \frac{1}{\sqrt{\ln(E/\varepsilon)}} \]

Elongation rate = 85 (g/cm²)/decade
Heitler toy model for electromagnetic showers

"Electron-photon" particle

Splitting length $\lambda$

Critical energy $\varepsilon$

\[ N(X, E) = 2^{X/\lambda} \]

\[ N_{\text{max}}(E) = \frac{E}{\varepsilon} \]

\[ X_{\text{max}}(E) = \lambda \log_2 \left( \frac{E}{\varepsilon} \right) \]
Electromagnetic showers:

\[ \langle X_{\text{max}}(E) \rangle = X_0 + D_\gamma \log E \]

\[ D_\gamma = \ln 10 \ X_{\text{rad}} \simeq 85 \ \text{g cm}^{-2} \]

Fluctuations:

\[ \sigma_X^2(\gamma, E) = \text{constant} \]

\[ \sigma_X^2(\gamma, E) \simeq 1.1 \ X_{\text{rad}} \simeq 40 \ \text{g cm}^{-2} \]
Proton Shower

Vertices: theoretically not understood (and energy dependent)

\[ \pi^0 \rightarrow \gamma \gamma \]

\[ \pi^+ \rightarrow \mu^+ \nu_\mu \]
HADRONIC INTERACTIONS

\[ p + A_{\text{Air}} \rightarrow p, n, \pi^0, \pi^\pm, K^\pm, K_L \ldots \]

Leading nucleon \(~50\%~\) of energy

\[ \pi^0 \rightarrow \gamma \gamma \]
Electromagnetic Shower

Inclusive spectra of secondary particles

Decay
\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]
\[ \downarrow \]
\[ e^+ + \nu_e + \bar{\nu}_\mu \]

Interaction
Toy Model for hadronic shower

\[ p + \text{air} \rightarrow \left( \frac{n}{2} \right) \pi^0 \rightarrow n \gamma \]

Energy equally divided among \( n \) photons.

\[ E_\gamma \approx \frac{E_0}{n} \]

\[ \frac{dN_\gamma}{dz} = \sum_n P_n \delta \left[ z - \frac{1}{n} \right]^n \]
\[ \langle X^{(p)}_{\text{max}} \rangle = \langle X_{1\text{st}} \rangle + X_{\text{rad}} \]
\[ \left\langle \log \left( \frac{E_0}{n_\gamma \varepsilon} \right) \right\rangle \]

1\textsuperscript{st} interaction

Development of photon shower of energy E/n
\[ \langle X_{\text{max}}^{(p)} \rangle = \langle X_{1\text{st}} \rangle + X_{\text{rad}} \left\langle \log \left( \frac{E_0}{n_{\gamma} \varepsilon} \right) \right\rangle \]

\[ \langle X_{\text{max}}^{(p)} \rangle = \lambda_p + X_{\text{rad}} \log \left( \frac{E_0}{\varepsilon} \right) \]

- Interaction Length
- Equivalent to photon shower
- Particle production properties
\[ \langle X_{\text{max}}^{(p)} \rangle = \lambda_p + X_{\text{rad}} \log \left( \frac{E_0}{\varepsilon} \right) - X_{\text{rad}} \langle \log n_\gamma \rangle \]

**Interaction length**

**“Softness”**

**Elongation Rate**

\[
\frac{d\langle X_{\text{max}}^{(p)} (E) \rangle}{d \log E} = X_{\text{rad}} + \frac{d\lambda_p (E)}{d \log E} - X_{\text{rad}} \frac{d\langle \log n_\gamma (E) \rangle}{d \log E}
\]

**Evolution with energy of the Interaction length**

**Evolution with energy of the “softness” of the spectrum**
Scaling model:
85 \text{(g/cm}^2)/\text{decade}

Increasing cross sections

Softer spectra

Elongation Rate for protons

Log[Energy]
Total pp Cross Section
Interaction Lengths (proton, pion)
Pion x-distribution at $10^{17}$ eV pp interaction

Artificial two examples

$\frac{1}{\sigma_{in}} \int x \frac{d\sigma}{dx}$

$\sigma_{in}$

$1$

$0.1$

$0.01$

$0.001$

$0.01$

$0.1$

$1$

$x_{cm}$

$10^{17}$ eV proton induced showers

Vertical shower

$N_e$ (Number of Electrons)

$10^8$

$10^7$

$10^6$

Vertical Depth (g/cm²)

$200$

$300$

$400$

$500$

$600$

$700$

$800$

$900$

$1000$

ad-hoc A

ad-hoc B
Phenomenological Evidence for SCALING

FERMILAB: \( pp \)

\[ \frac{d\sigma}{dx_F} \text{ (mbarn)} \]

\[ \begin{align*}
\bullet & \quad E_p = 100 \text{ GeV} \\
\circ & \quad E_p = 175 \text{ GeV}
\end{align*} \]
NUCLEAR effects: $pp$ vs $p^{-12}C$

NA49
EXTRAPOLATION to HIGH ENERGY (Pythia pp)

\[ E_{\text{beam}} = 10^{19}, 10^{17}, 10^{15}, 10^{13} \ \text{eV} \]

\[ \text{pp} \rightarrow \pi^+ \]

\[ z = \frac{E_{\text{lab}}}{E_{\text{beam}}} \]
EXTRAPOLATION to HIGH ENERGY (Pythia pp)

Large scaling violation for “central particles”

\[ E_{\text{beam}} = 10^{19}, 10^{17}, 10^{15}, 10^{13} \text{ eV} \]

\[ pp \rightarrow \pi^+ \]
EXTRAPOLATION to HIGH ENERGY (Pythia pp)

Small scaling violation [?] for “forward particles”
Charged pion spectrum

\[ E_0 = 10^{13}, 10^{15}, 10^{17}, 10^{19} \text{ eV} \]

**Sibyll**

**Montecarlo code**
PYTHIA PROTON Spectra

\[ E_{\text{beam}} = 10^{19} \text{ eV} \]

\[ z = \frac{E_{\text{lab}}}{E_{\text{beam}}} \]

[Graph showing proton and antiproton spectra]
PROTON Spectra  (elasticity spectra)

\[ E_{\text{beam}} = 10^{19}, 10^{17}, 10^{15}, 10^{13} \text{ eV} \]

\[ z = \frac{E_{\text{lab}}}{E_{\text{beam}}} \]

- Plot showing the differential cross section as a function of the parameter \( z \) for different beam energies.
PROTON Spectra (elasticity spectra)

pp → p

$E_{\text{beam}} = 10^{19} \text{ eV}$

$E_{\text{beam}} = 10^{13} \text{ eV}$
From Cosmic Ray Data → Hadronic Interactions

C.R. DATA

Astrophysical Information

“Astrophysical Composition Methods”

1 < A < 56 (very likely)

Hadronic Interactions
“Astrophysical Composition Methods”

- Energy Spectrum
  “imprints” of Energy Loss

- “Cosmic Magnetic Spectrometer”
IF one accepts (at least for the sake of discussion) the astrophysical hints of a proton dominated composition....
IF one accepts (at least for the sake of discussion) the astrophysical hints of a proton dominated composition....
If the highest energy CR are protons:
Models incorrect.
Need to make the showers shorter!

(higher cross sections, softer spectra)

If the highest energy CR are mostly iron:
Models incorrect.
Need to make the showers longer!

(smaller cross sections, harder spectra)
C.R. DATA

Astrophysical Information
- Energy Spectrum
- Composition

Hadronic Interactions
- Cross sections,
- Inclusive spectra
- Multiplicities
Let us measure the inclusive particle spectra at high energy in a laboratory!
..... but the measurements
have been performed ....
LHCf data hep-ex/1104.5294 [photon distributions in 2 very forward angular regions]
LHCf $\sqrt{s}=7\text{TeV}$

Gamma-ray like

$8.81 < \eta < 8.99$, $\Delta\phi = 20^\circ$
LARGE DISCREPANCIES !

What is the significance for the understanding of hadronic interactions?

What is the impact on the interpretation of UHECR?
Pseudo-Rapidity versus angle:

Very small angle production:
$\text{LHCf \sqrt{s} = 7\text{TeV}}$
\text{Gamma-ray like}

$8.81 < \eta < 8.99, \Delta\phi = 20^\circ$

\[
\left[ \frac{dN_\gamma}{dE_\gamma}(E_\gamma) \right]_{8.81 \leq \eta \leq 8.99} = \frac{dN_\gamma}{dE_\gamma}(E_\gamma) \times \frac{dN_\gamma[8.81 \leq \eta \leq 8.99]}{dN_\gamma[\text{all } \eta]}
\]

$\eta > 10.94, \Delta\phi = 360^\circ$

\[
\left[ \frac{dN_\gamma}{dE_\gamma}(E_\gamma) \right]_{\eta > 10.94} = \frac{dN_\gamma}{dE_\gamma}(E_\gamma) \times \frac{dN_\gamma[\eta > 10.94]}{dN_\gamma[\text{all } \eta]}
\]
\[
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\]

\[
\left[ \frac{dN_\gamma}{dE_\gamma}(E_\gamma) \right]_{\eta > 10.94} = \frac{dN_\gamma}{dE_\gamma}(E_\gamma) \times \frac{dN_\gamma[\eta > 10.94]}{dN_\gamma[\text{all } \eta]}
\]

Directly relevant for UHECR shower development

pT distribution dependence
\[ p_\perp = \sqrt{E^2 - m^2} \sin \left[ 2 \tan^{-1} (e^{-\eta}) \right] \]

\[ p_\perp \approx p \ 2 \ e^{-\eta} \]

\[ \eta > 10.94 \]

\[ 8.81 < \eta < 8.99 \]
\[ \eta > 10.94 \]
\[ 8.81 < \eta < 8.99 \]
\[ \int_{\eta_1}^{\eta_2} d\eta \frac{dN_\gamma}{dE_\gamma} \frac{dN_\gamma}{d\eta} \]

Rapidity distribution
[for a fixed energy]

\[ \frac{dN_\gamma}{dE_\gamma} \frac{dN_\gamma}{dp_{\perp}} \]

PT distribution
[for a fixed energy]
Transverse-Momentum Distribution

\[ \langle p_T \rangle = 0.5 \text{ GeV} \]
Transverse-Momentum Distribution

Tsallis parametrization
Used by CMS

\[
\frac{dN}{dp_T} \propto p_T \left(1 + \frac{E_T}{nT}\right)^{-n}
\]

\[
E_T = \sqrt{p_T^2 + m^2} - m
\]

\(<p_T> = 0.5 \text{ GeV}\)

Tsallis

Gaussian
Pseudo-Rapidity Distribution
(E = 1500 GeV)

$E = 1500$ GeV
$\langle p_T \rangle = 0.5$ GeV

$\frac{dN}{d\eta}$

$\eta$

$0.0$ $0.2$ $0.4$ $0.6$ $0.8$

$6$ $7$ $8$ $9$ $10$ $11$ $12$

Gaussian

Tsallis

$\langle p_T \rangle = 0.5$ GeV
Given a fixed Transverse-momentum distribution One obtains a pseudorapidity distribution

Factorized transverse momentum distribution: Translation of the same shape for pseudo-rapidity
$E = 0.5, 1, 1.5, 2, 2.5$ Te

$\langle p_T \rangle = 0.5$ GeV

Gaussian

$E = 0.5, 1, 1.5, 2, 2.5$ Te

$\langle p_T \rangle = 0.5$ GeV

Tsallis
Fraction of events in angular acceptance windows

Gaussian: $<p_T> = 0.5 \text{ GeV}$

$8.81 < \eta < 8.99$

$\eta > 10.94$
Fraction of events in angular acceptance windows

Solid: Gaussian
Dashed: Tsallis
Ratio \([\text{High pseudo-rapidity}] / [\text{Low pseudo-rapidity}]\)

Gaussian

\[\langle p_T \rangle = 0.4 \text{ GeV}\]
\[\langle p_T \rangle = 0.5 \text{ GeV}\]
\[\langle p_T \rangle = 0.6 \text{ GeV}\]
LHCf data

\[ \log \left( \frac{dN_{\gamma}}{dE_{\gamma}} \right) \]

$E_{\gamma}$ (TeV)

$8.81 < \eta < 8.99$

$\eta > 10.94$
Ratio \ [\text{High Rapidity}] / [\text{Low Rapidity}] 

for LHCf DATA
The pT distribution at $\sqrt{s} = 7$ TeV is not a Gaussian of energy independent width.
Ratio \ [\text{High Rapidity}] / \ [\text{Low Rapidity}]

![Graph showing the ratio of high and low rapidity data against energy (E_\gamma) for Pythia and Sibyll models.](image-url)
LHCf $\sqrt{s}=7$ TeV

Gamma-ray like

$\eta > 10.94, \Delta\phi = 360^\circ$

Common qualitative behaviour for ALL MC codes

"Curvature" in the ratio

With a minimum
LHCf $\sqrt{s}=7$ TeV

Gamma-ray like

$\eta > 10.94, \Delta \phi = 360^\circ$

Large difference between SIBYLL PYTHIA
Compare Pythia/Sibyll inclusive photon distribution:
Remarkably similar!  [pp interactions]
Compare Pythia/Sibyll charged pion distribution

\[ \frac{dN}{dE_{\gamma}} \]

\[ E_{\gamma} (\text{GeV}) \]

\[ \pi^+ \]

Solid: Pythia
Dashed: Sibyll
Compare the Energy flow
Pythia (photon/charged pion energy distributions)
Photon spectrum is softer
Average $p_T$ for photons for different energy regions

Pythia, $\gamma$
Pythia
Red: $\pi^+$
Blue: $\gamma$

$\langle p_T \rangle$ (GeV)

E (GeV)
pT of photon determined by pT of parent pion

\[
E_\pi >> m_\pi
\]

\[p_{T,\pi} = 0.5 \text{ GeV}\]
Average pT for photons and pions for different energy regions

SIBYLL MC code
Photons
Red: Pythia
Black: Sibyll
Pythia

**Photons**

- $E_\gamma = [0.1, 0.2 \text{ TeV}]$
- $E_\gamma = [0.2, 0.5 \text{ TeV}]$
- $E_\gamma = [0.5, 1.0 \text{ TeV}]$
- $E_\gamma = [1.0, 2.0 \text{ TeV}]$

Pythia

- $\pi^+ = [0.1, 0.2 \text{ TeV}]$
- $E_\gamma = [0.2, 0.5 \text{ TeV}]$
- $E_\gamma = [0.5, 1.0 \text{ TeV}]$
- $E_\gamma = [1.0, 2.0 \text{ TeV}]$
- $E_\gamma = [2.0, 3.5 \text{ TeV}]$
Photons

$E_\gamma = [0.2, 0.5 \text{ TeV}]$

Solid : Pythia
Dashed: Sibyll

$dN_\gamma/\,dp_T$

$p_T (\text{GeV})$

Photons

$E_\gamma = [1.0, 2.0 \text{ TeV}]$

Solid : Pythia
Dashed: Sibyll

$dN_\gamma/\,dp_T$

$p_T (\text{GeV})$
Most of the large discrepancies between Data and Montecarlo codes can be attributed to an incorrect modeling of the Transverse Momentum distributions.
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Without a good understanding of these Transverse Momentum distributions, it is not possible to interpret in a non-ambiguous way the LHCF published data for UHECR shower development.
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Can the LHCf instrument (together with other detectors at LHC) give more information on the pseudo-rapidity distributions?
We are studying at the same time

“Gigantic Astrophysical Beasts”
Millions of light years away
Length scale $10^{+24}$ cm

Microscopic
Partonic constituents of matter
Length scale $10^{-15}$ cm

Exciting
Difficult
$X_{\text{max}}$ fluctuations

Auger

HiRes
$X_{\text{max}}$ fluctuations

Auger

Quasi-monotonically Decreasing RMS

Constant RMS

HiRes
Overall comparison of Xmax data with QGSJET02 p and FE HIRES

Fig. 11.—Top: $X_{\text{max}}$ overlay of HiRes data (points) with QGSJET02 proton Monte Carlo airshowers after full detector simulation. Bottom: $X_{\text{max}}$ overlay of HiRes data (points) with QGSJET02 iron Monte Carlo airshowers after full detector simulation.
FLUCTUATIONS on \( X_{\text{max}} \)

\[
X_{\text{max}} = X_{1\text{st}} + Y_{\text{max}}
\]

\[
\sigma_{X_{\text{max}}}^2 = \sigma_{X_{1\text{st}}}^2 + \sigma_{Y_{\text{max}}}^2
\]

\[
\left( \sigma_{\langle X_{\text{max}} \rangle}^{\text{proton}} \right)^2 \approx \lambda_p^2 + \sigma_{Y_{\text{max}}}^2
\]

Toy model

\[
\left( \sigma_{\langle X_{\text{max}} \rangle}^{\text{proton}} \right)^2 \approx \lambda_p^2 + X_{\text{rad}}^2 \left[ \langle (\ln n_\gamma)^2 \rangle - \langle \ln n_\gamma \rangle^2 \right]
\]
\[
(\sigma_{X_{\text{max}}}^{\text{proton}})^2 \simeq \lambda_p^2 + \sigma_{Y_{\text{max}}}^2
\]

\[
(\sigma_{X_{\text{max}}}^{A})^2 \simeq f(A) \lambda_p^2 + \frac{\sigma_{Y_{\text{max}}}^2}{A}
\]

\[
A = 56
\]

\[
\frac{1}{\sqrt{A}} = 0.13
\]

\[
\sqrt{f(A)} \simeq 0.4
\]

Nuclear interaction.
Several Nucleons
Interact at same point.
$^{56}\text{Fe}$ interactions. $E_{\text{tot}} = 10^{19}$ eV

$\lambda_p = 48.5 \text{ g cm}^{-2}$

$\sigma\langle X_{1st} \rangle \approx \frac{\lambda_p}{\sqrt{A}} \approx 6.5 \text{ g cm}^{-2}$

$\sigma\langle X_{1st} \rangle \approx 20.5 \text{ g cm}^{-2}$
\[ \sigma_X^2 = \sum_j f_j \sigma_{A_j}^2 + \sum_j f_j \langle X_{A_j} \rangle^2 - \left( \sum_j f_j \langle X_{A_j} \rangle \right)^2 \]

\[ \sigma_X^2 = \langle \sigma_A^2 \rangle + D_p \left[ \langle (\log A)^2 \rangle - \langle \log A \rangle^2 \right] \]

\[ \sigma_X^2 \sim \langle \sigma_A^2 \rangle + D_p \sigma_{\log A}^2 \]
Mixing Protons with Iron-nuclei

\[ \sigma^2_X = f_p \sigma^2_p + (1 - f_p) \sigma^2_{Fe} + f_p (1 - f_p) \left( \langle X_p \rangle - \langle X_{Fe} \rangle \right)^2 \]

\[ f_{iron} = 1 - f_{proton} \]
THEORY

Construction of Hadronic Models
Hadronic Interactions

Composite (complex) Objects
Multiple interaction structure

QCD
“Cartoon” of a pp interaction in the transverse plane
Total Cross section

\[ \sigma_{\text{tot}} \quad \sigma_{\text{el}} \]

Properties of Particle Production

- Multiplicities
- Energy spectra
- \[ \frac{dN_{\text{ch}}}{dp_{\perp} \, dy} \]
Total Cross section

$\sigma_{\text{tot}} \quad \sigma_{\text{el}}$

Properties of Particle Production

Multiplicities
Energy spectra

...$
\frac{dN_{\text{ch}}}{dp_{\perp} \ dy}$

Larger Multiplicity
More “complex” events

Higher cross section
Elastic Scattering Amplitude:

\[
\frac{d\sigma_{el}}{dt}(t, s) = \pi \frac{d\sigma_{el}}{d^2q}(\tilde{q}, s) = \pi |F_{el}(\sqrt{-t}, s)|^2
\]

\[
F_{el}(q, s) = i \int \frac{d^2b}{2\pi} e^{i\tilde{q} \cdot \tilde{b}} \Gamma_{el}(b, s)
\]

\[
\Gamma_{el}(b, s) = 1 - e^{-\chi(b, s)}
\]
\[ \sigma_{el}(s) = \int d^2b |\Gamma_{el}(b, s)|^2 \]

\[ \sigma_{tot}(s) = 4\pi \text{Im}[F_{el}(0, s)] = 2 \int d^2b \text{Re}[\Gamma_{el}(b, s)] \]

\[ \sigma_{inel}(s) = \int d^2b \{1 - |1 - \Gamma_{el}(b, s)|^2\} \]

Total, elastic, inelastic cross section
Expressed in terms of the profile function
Total, Elastic, Diffractive Cross Sections:

1 minute of “19th century physics”: The OPTICAL ANALOGY.

Absorption and Scattering of light from an Opaque screen
\[ \sigma_{\text{el}} = \sigma_{\text{abs}} = \pi R^2 \]
Elastic scattering distributions

\[ \sigma_{\text{el}} = \frac{\sigma_{\text{tot}}^2 (1 + \rho^2)}{16\pi B_{\text{el}}} \]

\[ \sigma_{\text{abs}} = g \pi R^2 \]

\[ \sigma_{\text{el}} = g^2 \pi R^2 \]
Absorption profiles

Elastic scattering

\[ B = \frac{\langle b^2 \rangle}{2} \]
ISR 62.3 GeV
CERN UA4 546 GeV
“Absorption profile”
obtained from the elastic scattering of pp

ISR, CERN SpS (UA4), CDF

\[ \chi(b, s) \]

\[ \Gamma_{el}(b, s) \neq 1 - e^{-A(b) \sigma_{eik}(s)/2} \]

Failure of factorization
\[ \sigma_{\text{tot}}(s) = 4\pi \text{Im}[F_{\text{el}}(0, s)] = 2 \int d^2b \text{Re}[\Gamma_{\text{el}}(b, s)] \]

\[ \sigma_{\text{el}}(s) = \int d^2b |\Gamma_{\text{el}}(b, s)|^2 \]

\[ \sigma_{\text{inel}}(s) = \int d^2b \left\{ 1 - |1 - \Gamma_{\text{el}}(b, s)|^2 \right\} \]

Interaction Probability

\[ \Gamma_{\text{el}}(b, s) \equiv 1 - e^{-\chi(b, s)} = 1 - \sqrt{P_0(b, s)} = 1 - \exp \left[ -\frac{\langle n(b, s) \rangle}{2} \right] \]

"Interpretation" of the eikonal function

Multiple interactions
Identification of Eikonal function with
The average number of “elementary interactions”
At impact parameter $b$.

\[ \chi(b, s) = \frac{\langle n(b, s) \rangle}{2} \]

Cross section for “elementary interactions”

\[ \int d^2b \; \langle n(b, s) \rangle = \sigma_{\text{parton}}(s) \]
\[ \chi(b, s) = \frac{\langle n(b, s) \rangle}{2} \]

Identification of Eikonal function with the average number of “elementary interactions” at impact parameter \( b \).

Construction of \( \langle n(b, s) \rangle \)

Fluctuations of this average quantity.

Explicit construction of the final state at the “parton level”. 
Perturbative contribution to the Parton cross section

\[
\frac{d^3 \sigma}{dp_\perp dx_1 dx_2} \bigg|_{\text{jet pair}} (p_\perp, x_1, x_2; \sqrt{s}) = \sum_{j,k,j',k'} f_j^{h_1}(x_1, \mu^2) f_k^{h_2}(x_2, \mu^2) \frac{d\hat{\sigma}_{jk\rightarrow j'k'}}{dp_\perp}(p_\perp, \hat{s}).
\]

\[
\sigma_{\text{jet}}(p_{\perp}^{\text{min}}, \sqrt{s}) = \int_{p_{\perp}^{\text{min}}}^{\sqrt{s}/2} dp_\perp \int_{4p_\perp^2/s}^{1} dx_1 \int_{4p_\perp^2/(sx_1)}^{1} dx_2 
\left\{ \sum_{j,k,j',k'} f_j^{h_1}(x_1, \mu^2) f_k^{h_2}(x_2, \mu^2) \frac{d\hat{\sigma}_{jk\rightarrow j'k'}}{dp_\perp}(p_\perp, \hat{s}) \right\}
\]

\[p_{\perp}^{\text{min}} \rightarrow 0 \quad \text{Infrared Divergence} \quad \text{!!}
\]

\[\sigma_{\text{jet}} \rightarrow \infty \quad \text{(complete failure of perturbation theory)}
\]

Attempts to “resum” the soft part.
Parton Distribution Function

\[ Q^2 = 10 \text{ GeV}^2 \]

\[ x_f(x, Q^2) \]
MULTIPLE INTERACTIONS

- Estimate of the average number of Elementary interactions per pp scattering

- “Spatial Distribution” [proton spin] (Transverse coordinates) of the partonic constituents.

- Fluctuations of the “parton configuration” of an interacting hadron. Beyond PDF's Parton Distribution Functions
“Good-Walker ansatz” for inelastic diffraction.

[Extension of the optical analogy]

Scattering of polarized light from a “polarimeter”

Polarizing gray disk

\[
|x'\rangle = \cos \varphi |x\rangle + \sin \varphi |y\rangle, \\
|y'\rangle = -\sin \varphi |x\rangle + \cos \varphi |y\rangle
\]

Incident beam: \( |x\rangle \)

Absorption of \( |x'\rangle \)

Out scattered light

In polarizations

\( |x\rangle \) \hspace{1cm} \( |y\rangle \)

Elastic scattering

“inelastic diffraction”
Extension of the "Good-Walker Ansatz to the scattering of Hadronic Waves."

\[ |\varphi_m\rangle \quad \text{Observable states.} \quad |pp\rangle \quad |p\Delta\rangle \quad |p\Delta'\rangle \]
\[ |\Delta\Delta\rangle \quad |\Delta'\Delta\rangle \]

\[ |\psi_j\rangle \quad \text{"Transmission eigenstates"} \]
\[ T|\psi_j\rangle = t_j|\psi_j\rangle \]
\[ \text{["Parton Configuration states"]} \]
\[ \text{(Miettinen-Pumplin)} \]

2 orthonormal basis in Hilbert space

\[ |\varphi_m\rangle = \sum_i C_{m,i} |\psi_j\rangle \]
\[ |\psi_j\rangle = \sum_m C^*_{m,j} |\varphi_m\rangle \]
\[ t_j(b) = 1 - \exp \left[ - \frac{n_j(b)}{2} \right] \] One profile function for each "transmission eigenstate"

\[ \sigma_{\text{tot}} = \int d^2b \sum_j |C_{1j}|^2 2\text{Re}[t_j(b)] \]

\[ \sigma_{\text{abs}} = \int d^2b \left[ 1 - \sum_j |C_{1j}\{1 - t_j(b)\}|^2 \right] \]

\[ \sigma_{\text{diff+el}} = \sum_m \sigma_m = \int d^2b \sum_j |C_{1j}|^2 |t_j(b)|^2 \]
\[
\frac{d\sigma_{\text{abs}}}{d^2 b} = 1 - e^{-n(b)}
\]

\[
1 - \sum_j |C_{1j}|^2 e^{-n_j(b)}
\]

\[
1 - \int d\mathcal{C}_1 \int d\mathcal{C}_2 \; P_{h_1}(\mathcal{C}_1) \; P_{h_2}(\mathcal{C}_2) 
\exp \left[ -\frac{n(b, \mathcal{C}_1, \mathcal{C}_2)}{2} \right]
\]
Description of the “Underlying Event”

Qualitative result:

Events with 1 hard scatterings
Have more “activity” (larger multiplicity, ....)
Than the average event.

1. Select more “central” (lower b) interactions.

2. Select events where the colliding hadrons have certain “parton configurations”
(for example: more gluons in appropriate x interval)