Transverse momentum spectra of hadrons produced in Pb+Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV in the chemical non-equilibrium model

Viktor Begun

Jan Kochanowski University, Kielce, Poland

in collaboration with
Wojciech Florkowski and Maciej Rybczyński


Resonance Workshop at Catania, 3-7 November 2014
Motivation 1: Problems of thermal models with the proton yield

Statistical models have become one of the cornerstones of our understanding of heavy-ion and elementary \((e^+ e^-, p\bar{p})\) collisions. (Becattini, Braun-Munzinger, Broniowski, Cleymans, Gażdzicki, Gorenstein, Koch, Rafelski, Redlich, Satz, Stachel, Stock, ...)

The new data from LHC do not agree with the most common version of the thermal model for proton abundances.

Possible explanations:

- hadronic rescattering in the final stage
  (Becattini, Bleicher, Kollegger, Schuster, Steinheimer, Stock, PRL 111 (2013) 082302)

- hadronization and subsequent freeze-out taking place off chemical equilibrium

- incomplete list of hadrons
  (Noronha-Hostler, Greiner, 1405.7298, 1408.0761)

- flavor hierarchy at freeze-out
  (Chatterjee, Godbole, Gupta, PLB 727 (2013); Melo, Tomasik, QM 2014)
Besides the proton anomaly, the same LHC data exhibits another interesting feature:

**most central**

\[
\frac{1}{N_{\pi}} \frac{d^2 N_{\pi}}{d p_T^2} \text{ (GeV/c)}^2
\]

\[
\pi^+ + \pi^-
\]

\[
K^+ + K^-
\]

\[
p + p
\]

**semi central**

\[
\frac{1}{N_{\pi}} \frac{d^2 N_{\pi}}{d p_T^2} \text{ (GeV/c)}^2
\]

\[
\pi^+ + \pi^-
\]

\[
K^+ + K^-
\]

\[
p + p
\]

**peripheral**

\[
\frac{1}{N_{\pi}} \frac{d^2 N_{\pi}}{d p_T^2} \text{ (GeV/c)}^2
\]

\[
\pi^+ + \pi^-
\]

\[
K^+ + K^-
\]

\[
p + p
\]

The low-transverse-momentum pion spectra show **enhancement** by about 25%–50% with respect to the predictions of various thermal and hydrodynamic models

Motivation 2: Problems of hydrodynamic models with the pion spectra

IP - Glasma + MUSIC:

\[(1/2 / ) dN/dy p^T dp^T p^T [GeV]\]

ALICE
0-5%

Hydro with dynamical freeze-out:

AdS + hydro + cascade:

\(p^+ + K^+\)
\(K^+ + K^-\)
\(p + \bar{p}\)

\(p^+\) well described, protons?!

(Schee, Romatschke, Pratt, PRL (2013))
again pion enhancement!

(Huovinen et al., arXiv:1407.8152)

Viktor Begun (UJK)
November 3, 2014 4 / 20
1. Cracow single-freeze out model

Single-freeze out model (Broniowski, Florkowski, PRL 87 (2001) 272302)

The spectra are calculated from the Cooper-Frye formula at the freeze-out hyper surface

\[
\frac{dN}{dyd^2p_T} = \int d\Sigma \mu p^\mu f(p \cdot u), \quad t^2 = \tau_f^2 + x^2 + y^2 + z^2, \quad x^2 + y^2 \leq r_{\text{max}}^2,
\]

assuming the Hubble-like flow: \(u^\mu = x^\mu / \tau_f\).

There is only one additional parameter in the model, because the product \(\pi \tau_f r_{\text{max}}^2\) is equal to the volume (per unit rapidity), while the ratio \(r_{\text{max}} / \tau_f\) determines the slope of the spectra.

The phase-space distribution includes all well established resonances from PDG. The primordial distribution in the local rest frame has the form:

\[
f_i = g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\gamma_i^{-1} \exp(\sqrt{m^2 + p^2 / T}) \pm 1}, \quad \text{where} \quad \gamma_i = \gamma_q^N a_i + \gamma_s^N s_i \exp\left(\frac{\mu B_i + \mu S S_i}{T}\right),
\]

and \(N_q^i, N_s^i\) are the numbers of light \((u, d)\) and strange \((s)\) quarks in the \(i\)th hadron.
The parameters $\gamma_q$ and $\gamma_s$ are equivalent to the chemical potentials $\mu_i/T = \ln \gamma_i$

$$\gamma_i = \exp \left( \frac{\mu_q \left( N_i^q + N_i^{\bar{q}} \right) + \mu_s \left( N_i^s + N_i^{\bar{s}} \right)}{T} \right)$$

They are connected with the conservation of the SUM of the number of quarks and antiquarks during the hadronization process, similarly as $\mu_B$ and $\mu_S$ are connected with the conservation of the DIFFERENCE of the quark and antiquark numbers. (Rafelski: This must be so, since the entropy is conserved during the hadronization process.) This is valid when the hadronization process is fast and there is no significant volume expansion.

It can be also a result of the interplay between annihilation and recombination processes. For example, a $p\bar{p}$ annihilation to $n$ pions would produce the relation between nucleon and pion chemical potentials:

$$2 \mu_N = n \mu_\pi$$

Generally, $n$ may depend on energy, however in our case $n = 3$.

One can also imagine a QCD mechanism like the gluon condensation followed by the formation of low momentum $q\bar{q}$ pairs which fuse into pions which subsequently condense.
3. Spectra of pions, kaons and protons. Centrality dependence

Chemical non-equilibrium:
\[ V, T, \gamma_q, \gamma_s, r_{max}/\tau_f \]

Chemical equilibrium:
\[ V, T, r_{max}/\tau_f \]

One can observe a good agreement for pions and kaons, however protons in central collisions are described only in non-equilibrium.
3. Spectra of pions. Linear scale

**most central events**

![Graph showing pion spectra in most central events with chemical equilibrium and non-equilibrium conditions.]

- Primordial + secondary pions
- Secondary pions only

**Pb+Pb \( \sqrt{s_{NN}} = 2.76 \) TeV
- \( c = 0\% \div 5\% 

**semi central events**

![Graph showing pion spectra in semi central events with chemical equilibrium and non-equilibrium conditions.]

- Primordial + secondary pions
- Secondary pions only

**Pb+Pb \( \sqrt{s_{NN}} = 2.76 \) TeV
- \( c = 30\% \div 40\% 

Viktor Begun (UJK) November 3, 2014 8 / 20
4. Spectra of strange particles

Predictions for other hadrons:

The fit done initially for $\pi^+ + \pi^-$ and $K^+ + K^-$ only appears also very good for $p + \bar{p}$, $K_S^0$, $K^{*}(892)^0$ and $\phi(1020)$!
4. Spectra of strange particles. Hyperons

The possible sources of these discrepancies:

- **the thermodynamic parameters obtained when the data on multi-strange particles were not available**
- **unknown decays into $\Lambda$**
- **too much flow for heavy particles which is equivalent to the emission from a smaller volume in our model**
There is an upper bound on $\gamma_q$ and $\gamma_s$ because of Bose-Einstein condensation, the singularities may appear in the Bose-Einstein distributions of primordial pions and kaons. For pions, the value of $\gamma_s$ is irrelevant, and

$$\gamma_{\text{critic}}^q = \exp\left(\frac{m_{\pi^0}}{2T}\right)$$

Interestingly, the fits to the ratios of hadron abundances yield $\gamma_q$ which is very close to the critical value. It is equivalent to the pion chemical potential

$$\mu_\pi = 2T \ln \gamma_q \approx 134 \, \text{MeV}$$

which is very close to the $\pi^0$ mass, $m_{\pi^0} \approx 134.98 \, \text{MeV}$. It may suggest that a substantial part of $\pi^0$ mesons form the condensate. Therefore we add the estimation for the number of $\pi^0$ mesons $\pi^0 = (\pi^+ + \pi^-)/2$ in the $\chi^2$ fits with a condensate.

Large pion chemical potentials may lead to the formation of other types of condensates like a di-quark Bose condensate and many other phenomena.
5. Pion condensation

When chemical potential approaches to the mass of a particle, $\mu \to m$, the zero momentum level, $p_0 = 0$, and other low lying quantum states become important. Therefore one should make keep the summation over the low momentum states:

$$N = \sum_i \frac{g_i}{\exp\left(\frac{\sqrt{p_i^2 + m^2 - \mu}}{T}\right) - 1}$$

$$\to \frac{g_0}{\exp\left(\frac{m - \mu}{T}\right) - 1} + \frac{g_1}{\exp\left(\frac{\sqrt{p_1^2 + m^2 - \mu}}{T}\right) - 1} + \ldots + V \int_{p_j}^\infty \frac{d^3p}{(2\pi)^3} \frac{g}{\exp\left(\frac{\sqrt{p^2 + m^2 - \mu}}{T}\right) - 1}$$

One can show that the first and the last terms are proportional to $V$, while the second term is proportional to $V^{2/3}$ (Begun, Gorenstein, PRC (2008)). Therefore in the thermodynamic limit, $V \to \infty$, one may live only the $p_0 = 0$ term and start the integration from zero:

$$N \simeq \frac{g}{\exp\left(\frac{m - \mu}{T}\right) - 1} + V \int_0^\infty \frac{d^3p}{(2\pi)^3} \frac{g}{\exp\left(\frac{\sqrt{p^2 + m^2 - \mu}}{T}\right) - 1} = N_{\text{cond}} + N_{\text{norm}}$$

where $N_{\text{cond}}$ is the number of particles in Bose condensate and $N_{\text{norm}}$ is the number of particles in normal state.
5. Pion condensation. The volume and temperature

**EQ** - equilibrium model
**NEQ** - non-equilibrium model, previously shown
**BEC** - non-equilibrium with **condensate** at the zero momentum level - **NEW!**
Equilibrium model is not shown, since there $\gamma_q \equiv \gamma_s \equiv 1$

**NEQ** - non-equilibrium model, previously shown

**BEC** - non-equilibrium with **condensate** at the zero momentum level - **NEW!**
6. Conclusions

- The non-equilibrium thermal model combined with the single freeze-out scenario explains very well the spectra of pions, kaons, and protons.

- It eliminates the proton anomaly and explains the low-$p_T$ enhancement of pions.

- This enhancement may be interpreted as a signature of the onset of pion condensation in heavy-ion collisions at the LHC energies.

- It would be interesting to measure the pion spectrum at smaller values of $p_T$ than those available at the moment.

- The measurement of $\pi^0$ mesons is necessary to check for Bose condensation of pions.
Spectra of pions, kaons and protons at RHIC

Dariusz Prorok, Phys.Rev. C75 (2007) 014903, the same approach but applied for RHIC

chemical non-equilibrium  strangeness non-equilibrium  chemical equilibrium

situation opposite to that at the LHC!
Since we describe the spectra – the corresponding ratios are described automatically:
Spectra of strange particles. Hyperons

If the $\Sigma(1560)$ decay into $\Lambda$ is included and the $\Xi$’s and $\Omega$’s are emitted from a smaller volume, then the agreement is improved. However one should re-fit the new data before making conclusions.
Data analysis in SHARE thermal model


\[
\begin{array}{cccccccccccc}
\pi^+\pi^- & \frac{K^+K^-}{2} & K_0^0 & K^*+K^* & \phi & p+p & \Lambda & \Sigma^+\Sigma^- & \Omega^+\Omega^- & d & \frac{\lambda H^+\lambda H}{2} & \frac{\lambda He}{\lambda H} \\
\frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} & \frac{\phi}{\phi} \\
\end{array}
\]

ALICE Preliminary

\begin{align*}
Pb-Pb \; & \frac{\phi}{\phi} = 2.76 \; \text{TeV}, \; 0-10\% \\
\end{align*}

\[
\begin{array}{cccccc}
\text{Model} & T \; (\text{MeV}) & V \; (\text{fm}^3) & \gamma_s & \gamma_q & \chi^2/\text{NDF} \\
\hline
\text{SHARE 3} & 156 \pm 4 & 4364 \pm 848 & 1 (\text{fix}) & 1 (\text{fix}) & 12.4/6 \\
\text{SHARE 3} & 155 \pm 3 & 4406 \pm 766 & 1.07 \pm 0.05 & 1 (\text{fix}) & 9.6/5 \\
\text{SHARE 3} & 138 \pm 6 & 3064 \pm 1319 & 1.98 \pm 0.68 & 1.63 \pm 0.38 & 3.1/4 \\
\text{SHARE 3 (with nuclei)} & 152 \pm 8 & 4445 \pm 743 & 1.16 \pm 0.20 & 1.06 \pm 0.12 & 9.0/7 \\
\end{array}
\]

\[
\begin{align*}
\sigma (\text{mod.-data})/\sigma (\text{data}) & = 2.76 \; \text{TeV}, \; 0-10\% \\
\end{align*}

ALI-PREL-74481
Problems with the data

- **Different centrality selection for different data:**
  - \(\pi^\pm, K^\pm, p, \bar{p}\) and \(\phi(1020)\) are published in 10 centrality windows,
  - \(K^0_S\) and \(\Lambda\) are published in 7 centrality windows,
  - \(\Xi^\pm\) and \(\Omega^\pm\) are published in 5 centrality windows,
  - \(K^*(892)^0\) is published in 4 centrality windows.

- We found that the best way is to merge the data for \(\pi^\pm, K^\pm, p, \bar{p}\phi(1020), K^0_S\) and \(\Lambda\) to the centrality set of \(\Xi^\pm\) and \(\Omega^\pm\).

- The published combinations of particles are close, but do not coincide with those which one can calculate himself. For example, \(\Omega \neq \Omega^+ + \Omega^-\) and the ratio of \(\Lambda\) to \(K^0_S\) is different from the published \(\Lambda/K^0_S\) ratio, etc.

- At the moment we ignore the data on ratios and sums, because the usual \(\chi^2\) test should be done for the random variables, and not their combinations. However, it would be nice to find a way how to incorporate the data on ratios as well.