The Photon in Dense Nuclear Matter

The Equation of State of Neutron Star Matter, Catania 2017

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Photon spectrum in dense nuclear matter

Towards a realistic description including QED and strong interactions

1-loop (RPA) contributions:
- Landau damping
- Pair creation

2-loop contributions:
- Compton scattering
- N-N Bremsstrahlung

phenomenology

- Transport
- Damping of hydro/R modes
- Neutron star spin evolution
- Dark matter (dark photons)
RPA Photon Spectrum (relativistic)

One loop (pure QED) resummation

\[ \tilde{D}^{\mu\nu}(q) - D^{\mu\nu}(q) = \Pi^{\mu\nu}(q) \quad q = (q_0, \mathbf{q}) \]

- Dressed photon propagator in Coulomb Gauge:

\[ \tilde{D}^{\mu\nu}(q_0, q) = \frac{q^2}{q^2} (q^2 - \Pi_L)^{-1} P_L^{\mu\nu} + (q^2 - \Pi_\perp)^{-1} P_\perp^{\mu\nu} \]

Yields physical longitudinal and transverse spectrum incl. pair creation and Landau damping

\[ 2 \text{Im } \Pi = \sum \quad = \quad \sum \quad + \quad \sum \quad + \quad \sum \]

- In cold and dense matter \( \alpha_F \) corrections to the fermion propagator are tiny

[see e.g. textbooks of J. I. Kapusta or M. Le Bellac]

Regions where pair creation (PC) and Landau damping (LD) operate:

\[ \mu = 0 \text{ boundaries (left)} : \]
\[ q_0 = |q| \quad \text{(LD)} \]
\[ q_0 = \sqrt{|q|^2 + 4 m^2} \quad \text{(PC)} \quad \text{(right: dotted)} \]

\[ \mu - m \gg T \text{ boundaries (right)} : \]
\[ q_0 = -\mu + \sqrt{\mu^2 + |q|^2 \pm 2 k_f |q|} \quad \text{(LD, solid)} \]
\[ q_0 = +\mu + \sqrt{\mu^2 + |q|^2 - 2 k_f |q|} \quad \text{(PC, |q| > 2k_f, dashed)} \]
LD region: transport in NS cores

Transport coefficients in neutron star cores have been studied in detail

→ contributions from electron scattering important!
  (electrons are relativistic, degenerate, and weakly interacting)

→ collision energy and momentum transfer are usually assumed to be small!

→ Exchange of transverse photons dominant interaction mechanism!

- Weak screening approximation:

  \[ D_L \propto \frac{1}{q^2 - m_D^2} \] , \[ D_\perp \propto \frac{1}{q^2 - i \left( \frac{q_0}{|q|} \right) q_\perp^2} \] , \[ m_D^2 = \frac{4\alpha_f}{\pi} \mu k_f \] , \[ q_\perp^2 = \alpha_f k_f^2 \]

- HDL approximations: \(|q| \ll k_f\) not applicable for nucleons

GOAL: incorporate dynamical screening, correlations of QED + strong interactions
Single component (QED), real parts

Collective modes (poles of the resummed propagator)

\[ q^2 = Re \Pi_L(\omega, q), \quad \omega^2 = q^2 + Re \Pi_\perp(\omega, q) \]

Electron plasma

Proton plasma

Small q: \[ \omega^2_L = \omega_0^2 + \frac{3}{5} v_f^2 q^2, \quad \omega_\perp = c v_f |q|, \quad c \sim 0.83, \quad \omega_\perp = \omega_0^2 + \left(1 + \frac{1}{5} v_f^2\right) q^2 \]

“Plasma freq“: \[ \omega_0^2 = \frac{e^2 k_f^3}{3\pi^2 \mu} = \frac{1}{3} v_f^2 m_D^2 \]

\[ \mu_e = 122 \text{ MeV}, \mu_p = 589 \text{ MeV}, m_p^* = 575 \text{ MeV} \]
Single component (QED), imaginary parts

damping of modes

explains “thumb-like” shape of poles

\[ Im \Pi_L = -\frac{\pi}{2} m_D^2 \frac{\mu}{k_f |q|} q_0 \Theta(v_f |q| - q_0) \]

[HDL results: \((q_0, q) \ll k_f, \mu\)]


[M. Baldo, C. Ducoin, PRC 79, 035801 (2009)]
Single-component (QED), spectral function

spectrum due to LD and PC:

→ one-loop RPA leaves dissipation free region

\[ \rho_L(q) = \frac{1}{\pi} \frac{q^2}{q^2} \text{Im} \tilde{D}_L(q) = -\frac{1}{\pi} \frac{l}{[(R-q^2)^2-l^2]} |_{I \neq 0} + \text{sgn} \left( \frac{q_0}{|q|} \right) \delta(R - q^2) |_{\omega_L} \] (similar for \( \rho_\perp \))

\( R = Re \Pi_{00}, \quad I = Im \Pi_{00} \)

muon plasma \( |q| = 80 \text{ MeV} \)

\( |q| = 200 \text{ MeV} \)

\( \mu_\mu = 122 \text{ MeV} \)
Multi-component QED

Generalize to internal “flavour” space ($i \rightarrow e^-, \mu^-, p^+$)

$\Pi \rightarrow \text{diag}(\Pi_e, \Pi_\mu, \Pi_p)$, \hspace{5mm} $\gamma^\mu \rightarrow c^i \gamma^\mu$, \hspace{5mm} $c^i = (1, 1, -1)$

Dressed photon propagator:

\[
\widetilde{D}^{\mu\nu}(q_0, q) = \frac{q^2}{q^2} (q^2 - Tr [\Pi_L])^{-1} P_L^{\mu\nu} + (q^2 - Tr[\Pi_\perp])^{-1} P_\perp^{\mu\nu}
\]

- **Protons are quasiparticles** (strongly interacting Fermi liquid) with effective masses $m_p^*$
- **Collective modes** are oscillation *in the densities* of these quasiparticles

RPA resummation in multi-component plasma is well established:


Multi-component QED, real parts

Spectrum (electrons, muons, protons)

- Three damped modes \( (e, m, p) \)
  \( \omega_{<e}, \omega_{<\mu}, \omega_{<p} \)

- Two Bohm-Staver sound modes \( (m, p) \)
  [D. Bohm, T. Staver, Phys. Rev. 84, 836 (1950)]
  \( u_{\mu}, u_p \)

- One gapped (real) plasmon mode \( (e) \)
  \( \omega_L \)

- Transverse mode
  \( \omega_\perp \)

→ Light particles dynamically screen heavier ones.
Multi-component QED, imag. parts

Imaginary part (electrons, muons, protons)

Landau damping:
A \((e, m, p)\) \(\quad\) B \((e, p)\)
C \((e, m)\) \(\quad\) D \((p)\) \(\quad\) E\((e)\)

pair creation:
\(\bar{C} \ (e, m) \quad \bar{E} \ (e)\)

dissipation free modes:
\(\omega_L, \quad \omega_\perp\)

damped modes:
\(u_\mu, \quad u_p\)

over-damped modes:
\(\omega_{<,e}, \quad \omega_{<,\mu}, \quad \omega_{<,p}\)

\(n = n_0, \quad \mu_e = \mu_\mu = 122 \text{ MeV}, \quad \mu_p = 589 \text{ MeV}\)
Multi-component case, spectral functions

Static case:
\[ \Pi_L(q_0 = 0) = \frac{e^2}{\pi^2} \mu k_f , \]
\[ \Pi_\perp(q_0 = 0) = 0 . \]

spectrum determines:
→ scattering rates of leptons
→ transport
What's the role of the neutrons within RPA?

- Coupling to photon tiny in free space

- BUT: Interactions induced by the polarizability of protons
  

- use resummed RPA polarization tensor for protons
  
  \[ [S. Reddy, M. Prakash, J.M. Lattimer, J.A. Pons, PRC 59, 2888 (1999)] \]

\[ \Pi_p = \frac{\Pi_p (1 - V_{nn}\Pi_n)}{1 - V_{nn}\Pi_n - V_{pp}\Pi_p + (V_{nn}V_{pp} - V_{np}^2)\Pi_n\Pi_p} \]

\[ \bar{D}^{\mu\nu}(q_0, q) = \frac{q^2}{q^2} (q^2 - \Pi_{e,L} - \Pi_{\mu,L} - \bar{\Pi}_{p,L})^{-1} P_L^{\mu\nu} + (q^2 - \Pi_{e,\perp} - \Pi_{\mu,\perp} - \bar{\Pi}_{p,\perp})^{-1} P_{\perp}^{\mu\nu} \]

- Quasiparticle properties and interaction potentials \( V_{ij} \) obtained from Landau energy functional based on Skyrme type interactions
  
  \[ [N. Chamel, P. Haensel, PRC.73, 045802] \]
“Induced“ interactions

Nuclear interactions appear „nested“ inside electromagnetic ones

\[ V_{pp} = e + \mu + p + O(\alpha_f^2) \]

\[ = e + \mu + p + O(\alpha_f^2) \]

- \( V_{ab} \) are described by pointlike short-range (density- density and current-current) interactions:

\[
V_{\mu\nu}(q) = \frac{p_L^{\mu\nu} + p_\perp^{\mu\nu}}{q^2 + i\epsilon} \left( \begin{array}{cccc}
1 & 1 & -1 & 0 \\
1 & 1 & -1 & 0 \\
-1 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 \\
\end{array} \right) + \left( \begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
V_{pp} & V_{pn} & & \\
V_{np} & V_{nn} & & \\
\end{array} \right) g^{\mu0} g^{\nu0} + \left( \begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \bar{V}_{pp} & \bar{V}_{pn} \\
0 & 0 & \bar{V}_{np} & \bar{V}_{nn} \\
\end{array} \right) g^{\mu i} g^{\nu j}
\]
induced (static) screening

\[
\tilde{\Pi}_{L,p}(q_0 = 0) = \left[ \frac{\partial \mu_p(\mu_n)}{\partial n_p} \right]^{-1} = \frac{m_p^2(1 + V_{nn}m_n^2)}{1 + V_{nn}m_n^2 + V_{pp}m_p^2 + (V_{nn}V_{pp} - V_{np}^2)m_n^2m_p^2}
\]

→ Strong qualitative impact of induced interactions in any Skyrme parameter set (NRAPR, SKDEv01, SKRA, SQMC700, LNS)

→ To be combined with the dynamical screening effects from QED to yield realistic spectrum
Collective modes (QED + strong int.)

$n = 0.85 \, n_0$

induced int.

$n = 1.6 \, n_0$

compare to: [M. Baldo, C. Ducoin, PRC 79, 035801 (2009)]
Spectral function (QED + strong int.)

\[ n = 0.65 \, n_0, \quad |q| = 10 \, \text{MeV} \]

\[ n = 1.6 \, n_0, \quad |q| = 10 \, \text{MeV} \]

- Changes to transverse spectrum are negligible as \( \Pi_\perp \ll \Pi_L \).

\[ L_{\gamma-n} = e^2 \, V_{np} \left( \bar{n} \gamma_\mu \, n \right) \, A_\nu \left( \Pi_{L,p} P_\mu^{\nu} + \Pi_{\perp,p} P_\perp^{\mu
u} \right) \]

- Impacts the hypothesis that scattering by transverse photons is dominant (depending on density) [H. Heiselberg, G. Baym, C. J. Pethick, J. Popp, Nuc. Phys. A 544 (1992)]
Where to go from here:

- Use photon spectrum to refine existing calculations of transport coefficients in neutron star cores.
  [work in progress: E. Rrapaj, S. Reddy, S. Stetina]
- Improve the implementation of nuclear interaction potentials.
- Study the impact of pairing

The physics of dynamical screening and correlations of electromagnetic and strong interactions are important in the determination of transport in NS cores.
Outlook (II): high energy spectrum, dark matter?

- beyond one-loop (RPA): what happens in diss. free region?
  [work in progress: E. Rrapaj, S. Reddy, S. Stetina]

  - When one-loop results are kinemat. forbidden, two loop processes take over
  - nucl. int. extracted from the measured nucleon-nucleon elastic cross section
  - Study T dependence of processes (supernovae)

2 loop QED (e^4): Compton scatt.
2 loop QED + strong (e^2 V_{ab}): n-n Bremsstrahlung
mille grazie!