Collective excitations in the neutron star crust

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One of the neutron star observables is the surface temperature which can give constraints on the thermal evolution estimating its age.

- Specific heat is one of the elements to study the thermal evolution of neutron star.
- Specific heat is a sum over different contributions from the different excitations (nuclei, phonons, electrons,...)
- Shortly after the birth the core contains still a lot of energy which escapes through the crust $\implies$ I will study thermal properties of the crust.

**Figure:** Specific heat contribution as a function of the density at $T = 10^9 K$, thanks to Morgane Fortin.
We will be interested in the inner crust which contains the structure called "pasta phase".

**Figure**: Neutron star crust

- This part of the crust is characterised by the transition from homogeneous matter to the lattice of atomic nucleis.
- Pasta phase = very deformed nuclei.
What are the different contributions to thermal properties?

- To find all contributions to specific heat in the crust
- Paired nucleons: contribution strongly suppressed due to pairing gap.
- But superfluidity $\Rightarrow$ low energy collective excitations called hydrodynamic modes.
- First order perturbations in density and propagate at sound velocity.
- Our approach allows wave length larger than Wigner-Seitz cells.

Figure: Energy gap of pairing.
Description of the model:

- I take a simple model with 1D geometry
- This geometry correspond to a periodic alternance of two slabs ("gaseous" and "liquid") with different proton and neutron densities
- Zero temperature approximation ⟺ neutrons and protons are treated as superfluids.
- Superfluid hydrodynamics approximation in each slab
- Non-relativistic approximation.

Figure: Representation of "lasagna"
Two basic equations for deriving superfluid hydrodynamics:

- Conservation of particle number: \( \partial_\mu n^\mu = 0 \)
- Energy-momentum conservation (Euler equation): \( \partial_\mu T^{\mu\nu} = 0 \) with \( T^{\mu\nu} = P \delta^{\mu\nu} + \sum_{x=n,p} n_x^{\mu} \mu_x^{\nu} \)

Characteristics of hydrodynamics with two superfluid components (n,p):

- No viscosity.
- Entrainment between the two fluids: non dissipative interaction which misalign velocities and momentum.

Parameters appearing in the hydrodynamic equations are calculated within a Landau-Fermi liquid model. Relativistic Mean Field interaction with \( \sigma - \omega - \rho - \delta \) mesons is employed with density dependent parameters defined in Avancini et al. (2009).
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2. Boundary conditions between slabs:
   \[ \text{At } T \sim 10^9 \text{K} \Rightarrow \text{time period of modes } \ll \text{characteristic time of } \beta\text{-interaction and relaxation times. } \]
   \[ \Rightarrow \text{Fluids are impenetrable } \]
   \[ \Rightarrow \text{Contact is maintained } \]
   \[ \Rightarrow \text{Continuity of perpendicular fluid velocities and continuity of chemical potentials} \]
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4. We use the Floquet-Bloch theorem to take into account the periodicity
   \[ (U(\vec{r} + \vec{L}) = U(\vec{r})e^{i\vec{q} \cdot \vec{L}} \text{ where } \vec{L} \text{ is the periodicity}) \]
Two examples of chemical potential fluctuations through 3 slabs. The phase of the order parameter can be deduced with the Josephson relation

\[ \frac{\partial \phi^A}{\partial t} = -\delta \mu^A \]  

with \( A = n, p \)

**Figure**: Real part of the chemical potential fluctuations along \( z \) axis for lasagna at \( n_b = 0.0804 fm^{-3} \sim \frac{\rho_0}{2} \) and \( \omega = 10 MeV \) (left) and for droplets at \( n_b = 0.0013 fm^{-3} \) and \( \omega = 1 MeV \) (right).
Two examples of excitations spectrum for different angles of propagation.

Figure: Excitation spectrum at $n_b = 0.0804 fm^{-3} \sim \rho_0 / 2$ (left) and at $n_b = 0.0013 fm^{-3}$ (right).
Taking Bose distribution for this hydrodynamics modes we can integrate over all momenta in order to obtain the specific heat contribution.

**Figure**: Specific heat as a function of $T$ at $n_b = 0.0804\, fm^{-3} \sim \frac{\rho_0}{2}$.

**Figure**: Deviation of the specific heat from the $T^3$ law at $n_b = 0.0013\, fm^{-3}$.
Figure: Specific heat contributions of neutrons as a function of the density at $T = 10^9\,\text{K}$, thanks to Morgane Fortin
Figure: All specific heat contributions as a function of the density at $T = 10^9 \text{K}$, thanks to Morgane Fortin
I have introduced a formalism for wave propagation in 1D structures taking into account superfluidity and the periodicity. The dispersion relations show interesting acoustic and optic branches. The calculation of the specific heat, which follow a classical temperature dependance for bose distribution at high density and a deviation arround 10% at less density.

Perspective:

- Extend the model for other geometrical structures.
- Introduce Coulomb interaction
- Estimate the effect of this specific heat to the thermal evolution of the neutron star
- Non-zero temperature $\Rightarrow$ addition of a ”normal fluid”. 