Investigation of symmetry breaking and fluctuations on matter self-organisation in Neutron Star crusts, from crystalline to exotic structures

V. de la Mota, F. Sébille and S. Figerou
Subatech
France
Neutron star crust
At subnuclear densities

Nucleons correlated at short distance / **nuclear attraction**
- nucleons bind into nuclei

Nucleons anticorrelated at long distances / **Coulomb repulsion**.
- Coulomb crystal lattice

\[ \rho < \rho_0 \approx 2.1017 \text{ kg/m}^3 \]

The competition leads to

Frustrated Baryonic matter
- Very neutron rich nuclei
- inhomogeneous nuclear matter
- neutron superfluid

Characterized by

**Structural transitions**
Between exotic nuclear structures
**Structural transitions** under **Frustration**

- Energy minimisation
  - Minima determination

**Complementary strategies**

**Statics**

- Simulate Annealing
  - $\approx$ HF
  - $\approx$ Bloch functions

**Dynamics**

- « Dissipative »
  - E-TDHF

**N-body System**

- Self-organisation

**Wavelet representation**
Wavelet Representation

DYWAN Model

- Physical Formulation
  - Consistency
  - Projection Methods
    - Decomposition on orthogonal Subspaces
    - Relevant informations
    - Hierarchy of approximations
    - Scale decomposition
    - Symmetries, geometry
    - Entropy criteria

- Mathematical Representation
Wavelet transforms

\[ \Psi_{a,b}(x) = U(a,b) \Psi(x) = \frac{1}{\sqrt{a}} \Psi[(x-b)/a] \]

Wavelet Family
transforms
Analysing Wavelet

Theory of Group Representation
dilation translation

Statics
Wave function
Optimal representation

Dynamics
Variational principle

Initial Conditions depending
- Nuclear species, Crystal structure, Gas & dripped nucleons, temperature,

Dynamics driven by a Mean Field Evolution (TDHF)
- Nuclear effective interactions
- the long range Coulomb force

The Cooling/dissipation occupation number Master equation of the one body wave functions
Initial conditions: for cubic lattices

Periodic boundary conditions

Primitive Cell

Super-cell
Protonic Fraction \( x = 0.2 \)

\[ \langle \rho \rangle = 0.4 \rho_0 \]

**lattice:**
Cubic Simple CS

Oxygen Initial Nuclei

without lattice sites Perturbations

\[ S/K_B \]

\[ t \ (fm/c) \]
lattice: Cubic Simple CS

Fraction protonique $x=0.5$

$\Rightarrow$ Symmetry breaking

$\Rightarrow$ Entropy conservation

$\langle \rho \rangle = 0.4 \rho_0$

$S/K_B$

$0 \quad 100 \quad 200 \quad 300 \quad 400 \quad 500 \quad 600 \quad 700 \quad 800$

t (fm/c)
2D Density distribution

threshold density $\rho_t$:

corresponds to a plane cut $\rightarrow$

morphological structure

A unique average neutron density can display various morphological structures depending on the threshold density $\rho_t$. 

Threshold density $\rho_t$

Average density $\langle \rho \rangle$

Density isosurface

Average density $\langle \rho \rangle / \rho_\infty$

$\Omega = -100 \text{ MeV}, \chi = 0.5$

The structure diagram in two dimensions
Protonic Fraction $x=0.2$

Sponge like structures
Lower proton fraction and greater average densities

$\langle \rho \rangle = 0.9 \rho_0$
**lattice:**
Face Center Cubic (FCC)

Protonic Fraction $x=0.2$

**Oxygen Initial Nuclei**
**Protonic Fraction**

$x=0.5$

**Noticeable differences:**
- Few cylinders
- Greater uniform distribution
- Effect of Coulomb interaction

$x=0.3$ / Oxygen initial lattice

*similar behavior*
- Weaker influence of initial nuclei species
Multiple Perspectives

- Sensitivity / crystal structure
- EOS & Isospin
- Fluctuations
- Finite size effects and scaling laws
- **Transport properties:** viscosity
  thermal conductivity, shear modules,
  neutrino opacity, caloric capacity, ..
- super-fluidity
- Cooling dynamics
- Strong magnetic fields
$\psi(\rho_t) = \frac{\text{volume}}{\text{surface}} \approx 0$

$\rho_t^{\text{ref}}$ Reference threshold

Not always defined

**Effective Interactions**

**Zero-range Skyrme type forces**

\[
\omega(\rho, \delta) = \frac{t'_{10}}{2\rho_{\infty}} \rho + \frac{t'_{2}}{(v+2)\rho_{\infty}^{v+1}} \rho^{v+1} + \frac{c}{\rho_{\infty}^{2}} \delta^{2} \rho^{2} + \frac{\Omega}{3\rho_{\infty}^{2}} (\rho - \rho_{\infty}) \rho \delta^{2} + \omega_{\text{kin}}
\]

Energy per baryon

asy-stiff

asy-soft

Neutron matter

**Finite-range Gogny type forces**

These Effective forces are currently in implementation, wavelets providing a very efficient tool for gaussian finite ranges
Neutron structure diagram
In Isospin Asymmetric Matter

the influence of the EOS is more perceptible

as the force softens (for $\Omega$ increasing values), the slab formation increases and the sponge-like region decreases.

neutron chemical potential in pure neutron matter is stronger for the stiffest effective interaction

increase of sponge-like structures, in correlation with a greater spatial extension of the neutron liquid, linking the residual clusters
Behaviors similar to FCC lattices
Uniform distribution reached for greater average densities

Protonic Fraction $x=0.5$

Protonic Fraction $x=0.2$
primitive

Lattice: Cubic Simple (CS)

Iron

Initial Nuclei

Without lattice sites Perturbations

Protonic Fraction $x = 0.3$

Lattice: Face Center Cubic (FCC)

Perturbations

Protonic Fraction $x = 0.3$

$\langle \rho \rangle = 0.1 \rho_0$

$\langle \rho \rangle = 0.3 \rho_0$

$\langle \rho \rangle = 0.6 \rho_0$
Physics of exotic nuclei and nuclear matter

• What is nature of the nuclear force that binds protons and neutrons into stable nuclei and rare isotopes?
  • What are the nuclear reactions that drive compact stars and stellar explosions?

Common interest in nuclear physics and astrophysics

• Exploration of densities away from saturation one ρ0
  • Exploration of Large isospin asymmetries

Common methodology

isospin “diffusion”:

Heavy-ion collisions

Isospin transport phenomena

isospin “drift”: