Breaking stress of neutron star crust

A.I. Chugunov\textsuperscript{1}, C.J. Horowitz\textsuperscript{2}

\textsuperscript{1} Ioffe Institute (St. Petersburg, Russia)

\textsuperscript{2} Indiana University (Bloomington, Indiana, USA)
Talk overview

1. Introduction
2. Kinetic theory of strength
3. Molecular dynamics
4. The breaking stress
5. Some microphysical details
6. Results and conclusions
Introduction: motivation

How strong is the neutron star crust?

Breaking of crust: 

• Pulsar glitches
• Magnetar flares
• Gravitational waves from “mountains”

Approaches:

• Theory: Long-range interaction, Many body
  \[ \tau \sim \omega_p^{-1} \sim 10^{-21} \text{ c} \Rightarrow 10^{21} \text{ MD steps} \]

• Numerical: 1 MD step – 10 seconds per core

(Semi) phenomenology
Specifics of neutron star crust

• Interionic interaction:
  ➢ Long range
  ➢ Isotropic
  ➢ Soft core

• Almost uniform electronic background
  ➢ Almost perfect crystal
  ➢ Not cracks
First time direct simulations


Breaking strain $\epsilon \sim 0.1$
Aim
What is the lifetime of a statically deformed crust matter (at given stress, density, temperature and composition)?

Hypothesis

• Stress define probability of breaking event per unit time
• This probability depend on the temperature and defines lifetime (durability) of a crystal at given stress

Crust durability: Defined by the weakest point of the crust – the point with lowest durability at given distribution of stress
Glacier Calving

http://www.dailymotion.com/video/x29opv_glacier-calving-glacier-bay-alaska_travel

COMPSTAR 2011
Catania, Italy, 09-12 May 2011
Kinetic theory of strength


\[
\frac{\tau}{\tau_0} = \exp \left( \frac{U - \sigma V}{T} \right)
\]

Stress doesn’t cause breaking as it is, but breaking is result of thermal fluctuations
Kinetic theory of strength

Dimensionless variables

\[ t = t \omega_p, \quad \eta = V n_i \quad \mu = U \frac{a}{Z^2 e^2}, \quad \varsigma = \sigma \frac{a}{n_i Z^2 e^2} \]

\[ t = \exp (\mu \Gamma - \varsigma \eta \Gamma) \]

Linearly increasing stress (\( \sigma \propto t \))

\[ t = \varsigma \eta \Gamma \exp (\mu \Gamma - \varsigma \eta \Gamma) \]
Numerical: Molecular dynamics

\[ U_{jk} = \frac{Z_j Z_k e^2}{r_{jk}} \exp \left( -\frac{r_{jk}}{\lambda_e} \right) \]

\[ \left\{ r_j^{(l)}, V_j^{(l)} \right\} \]

\[ r_j^{(l+1)} = r_j^{(l)} + V_j^{(l)} \Delta t + a_j^{(l)} \frac{\Delta t^2}{2} \]

\[ F_{jk}^{(l+1)} = -\nabla r_j U_{jk}^{(l+1)} \]

\[ a_j^{(l+1)} = \frac{1}{m_j} \sum_{k \neq j} F_{jk}^{(l+1)} \]

\[ V_j^{(l+1)} = V_j^{(l)} + a_j^{(l)} + a_j^{(l+1)} \frac{\Delta t}{2} \]
Breaking stress in MD

Time-dependent periodic boundary conditions

\[ x \rightarrow x + \epsilon y/2 \]
\[ y \rightarrow y + \epsilon x/2 \]
\[ z \rightarrow z/(1 - \epsilon^2/4) \]
\[ \epsilon = vt \]

\[ \sigma = dU/d\epsilon \]
During the deformation:
The breaking stress

\[ \varepsilon = \nu t \]

\[ \mathcal{U} = 0.366 \]

\[ \mathcal{N} = \frac{500}{\Gamma - 149} + 18.5 \]

\[ \sigma_b^{\text{max}} = \left( 0.0194 - \frac{1.25}{\Gamma - 72} \right) n_i \frac{Z^2 e^2}{a} \]

\[ Z = 29.4, \quad \lambda_e/a = 1.75 \]
The breaking stress

\[ Z = 100, \frac{\lambda_e}{a} = 1.16 \]

\[ Z = 6, \frac{\lambda_e}{a} = 2.97 \]

\[ \mathcal{U} = \frac{68.6}{\Gamma_m}, \quad \mathcal{N} = \frac{500}{\Gamma - 0.8\Gamma_m} + 18.5 \]

\[ \Gamma_m = \Gamma_m^0 \frac{\exp(\kappa)}{1 + \kappa + \kappa^2/2}, \quad \kappa = \lambda_e n_i^{1/3} \]

Vaulina et al., PRE 66, 016404 (2002)
Correction for zero-point oscillations

Can not be studied directly by classical MD

Model: For order of magnitude the durability is defined by the root-mean-square displacement of ions

\[ \tilde{\Gamma} = \Gamma \left[ 1 + \frac{1}{4} \frac{T_p^2 u_1^2}{T^2 u_{-1}^2} \right]^{-1/2} \approx \Gamma \left[ 1 + 0.013 \frac{T_p^2}{T^2} \right]^{-1/2} \]

\[ t = \exp \left( \mu \tilde{\Gamma} - \varsigma \mathcal{N} \tilde{\Gamma} \right) \]
Breaking at realistic timescales

\[ \frac{\sigma_b}{P_e} \]

\[ T = 1 \times 10^6 \text{ K}, \Gamma = 4 \times 10^4, T/T_p = 0.009 \]

\[ T = 2 \times 10^7 \text{ K}, \Gamma = 2010, T/T_p = 0.17 \]

\[ T = 5 \times 10^7 \text{ K}, \Gamma = 801, T/T_p = 0.43 \]

\[ T = 1 \times 10^8 \text{ K}, \Gamma = 402, T/T_p = 0.87 \]

\[ \rho = 10^0 \text{ g cm}^{-3} \]

\[ T = 2 \times 10^8 \text{ K}, \Gamma = 201, T/T_p = 1.74 \]

\[ ^{56}\text{Fe} \]

\[ \sigma_b / [\text{a}, Z^2] e^2 / a \]
Breaking at realistic timescales

\[ \sigma_b / \left( n_i Z^2 e^2 / a \right) \]

- \( T = 1 \times 10^6 \text{ K}, \Gamma = 4 \times 10^4, T/T_p = 0.009 \)
- \( T = 2 \times 10^7 \text{ K}, \Gamma = 2010, T/T_p = 0.17 \)
- \( T = 5 \times 10^7 \text{ K}, \Gamma = 801, T/T_p = 0.43 \)
- \( T = 1 \times 10^8 \text{ K}, \Gamma = 402, T/T_p = 0.87 \)

\[ \rho = 10^9 \text{ g cm}^{-3} \]

\[ {^{56}\text{Fe}} \]

\[ T = 2 \times 10^8 \text{ K}, \Gamma = 201, T/T_p = 1.74 \]

\[ \Gamma = 1600 \]
\[ \Gamma = 3200 \]
\[ \Gamma = 800 \]
\[ \Gamma = 400 \]
\[ \Gamma = 300 \]
\[ \Gamma = 250 \]
\[ \Gamma = 200 \]
\[ \Gamma = 180 \]

\[ v / \omega_p \]

COMPSTAR 2011
Catania, Italy, 09-12 May 2011
Breaking event

$\Gamma = 800, \ (\omega_p \nu)^{-1} = 1.28 \cdot 10^5$

\[ \epsilon = 0.1234 \]

\[ \epsilon = 0.1238 \]
Before and after breaking

\[ \Gamma = 180, \; (\omega_p v)^{-1} = 1.28 \cdot 10^5 \]

\[ \epsilon = 0.0823 \]

\[ \epsilon = 0.0831 \]
Result and conclusions

1. We performed many molecular dynamic simulations to define the breaking stress at different coupling parameters (inverse temperatures), strain rates and composition of matter.
2. The results are in good agreement with the kinetic theory of strength. We extract parameters of this model from MD simulations, providing thus a simple approximation for the durability of neutron star crust matter.
3. We estimate the breaking stress for timescales $1 \text{s} - 1 \text{ year}$. The breaking stress decreases approaching the melting point.
4. There are a lot of issues to be addressed in the future (plasticity, for example).