Many-body theory of neutron star structure and dynamics

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Catania, May 12, 2011
Outline

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- Transport coefficients of neutron star matter
- The paradigm of *ab initio* many-body approaches
- Effective interactions derived from realistic nuclear hamiltonians
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  - Neutrino mean free path
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Motivation

★ The description of neutron star structure and dynamics (mass, radius, moment of inertia, gravitational wave emission, cooling mechanisms) requires the knowledge of a variety of nuclear matter properties, including the equation of state (EOS), the transport coefficients (viscosity, thermal conductivity, ... ) and the neutrino emission and scattering rates.
The description of neutron star structure and dynamics (mass, radius, moment of inertia, gravitational wave emission, cooling mechanisms) requires the knowledge of a variety of nuclear matter properties, including the \textit{equation of state} (EOS), the \textit{transport coefficients} (viscosity, thermal conductivity, \ldots) and the \textit{neutrino emission and scattering rates}.

While the EOS can be obtained from refined many-body calculations based on realistic dynamical models (see Arnau’s talk), most available estimates of other quantities are based on \textit{crude simplifying assumptions}.
Motivation

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★ In March 2010, Luciano Rezzolla urged the members of the COMPSTAR collaboration “to team up to produce a description of the matter properties of compact stars which is as complete as possible, thus providing not only a prescription for pressure vs density, but also all the important quantities such as viscosity, emissivity, conductivities, etc”.
Consider charge-neutral matter in $\beta$-equilibrium at baryon number density $n_B$. 
Nuclear Matter Equation of State

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★ Nuclear many-body theory (NMBT) provides a scheme to obtain the ground state per baryon for any values of the proton fraction $x$, thus allowing to determine the EOS at $T = 0$ (see Arnau’s talk)

$$e(n_B, x) = \frac{E}{N_B} \Rightarrow \epsilon(n_B) = n_B e, \quad P(n_B) = n_B^2 \left( \frac{\partial e}{\partial n_B} \right) \Rightarrow P = P(\epsilon)$$

with $x$ determined by the requirements charge neutrality and chemical equilibrium
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- Can the theoretical approaches used to compute the EOS be exploited to consistently obtain other important properties of neutron star matter?
Abrikosov & Khalatnikov (AK) formalism (AD 1957). Starting point: Boltzmann equation

\[
\frac{\partial n}{\partial t} + \frac{\partial n}{\partial r} \frac{\partial \epsilon_p}{\partial p} - \frac{\partial n}{\partial p} \frac{\partial \epsilon_p}{\partial r} = I(n)
\]

\[n = n_0 + \delta n, \quad n_0 = \{1 + \exp[\beta(\epsilon - \mu)]\}^{-1}\]
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★ The collision integral \(I(n)\) depends on the probability of the scattering process \(1 + 2 \rightarrow 1' + 2'\)

★ Consider shear viscosity as an example. Using Landau theory of Fermi liquids AK obtain the approximate (although rather accurate) result

\[
\eta_{AK} = \frac{1}{5} \rho m^* v_F^2 \tau \frac{2}{\pi^2(1 - \lambda_\eta)}
\]
quasiparticle lifetime and angle-averaged scattering probability $\langle W \rangle$

$$
\tau T^2 = \frac{8\pi^4}{m^*^3} \frac{1}{\langle W \rangle} \\
\langle W \rangle = \int \frac{d\Omega}{2\pi} \frac{W(\theta, \phi)}{\cos \theta/2}
$$

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exact solution by Brooker & Sykes (AD 1968)

$$\eta = \eta_{AK} C(\lambda_\eta)$$

$$C(\lambda_\eta) = \sum_{k=0}^{\infty} \frac{4k + 3}{(k + 1)(2k + 1)[(k + 1)(2k + 1) - \lambda_\eta]}$$

$$-2 < \lambda_\eta < 1 \quad , \quad 0.750 < C(\lambda_\eta) < 0.925$$
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\[ C(\lambda_\eta) = \frac{1 - \lambda_\eta}{4} \sum_{k=0}^{\infty} \frac{4k + 3}{(k + 1)(2k + 1)[(k + 1)(2k + 1) - \lambda_\eta]} \]

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Similar expressions can be obtained for the other transport coefficients
Calculations of transport coefficients

- Calculation of the transport coefficients within the AK approach requires
  - The quasiparticle spectrum $\epsilon_p$, needed to calculate the effective mass from
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    \frac{1}{m^*} = \frac{1}{p} \frac{d\epsilon_p}{dp}
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▶ The scattering probability, related to the scattering cross section in the nuclear medium through

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can only be obtained from dynamical models providing a description of neutron-neutron scattering processes. Mean field approaches fail to fulfill this requirement.
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- Strategy: use the available \textit{ab initio} approaches to obtain \textit{effective interactions}, derived from realistic nucleon-nucleon potentials, allowing for a consistent calculation of all relevant quantities.
The paradigm of *ab initio* non relativistic approaches

- Non relativistic pointlike protons and neutrons interacting through the hamiltonian

\[ H = T + V = \sum_i \frac{p_i^2}{2m} + \sum_{j>i} v_{ij} + \ldots \]
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- the resulting hamiltonian, involving no adjustable parameters, can be used to obtain the EOS within any of the theoretical approaches discussed in Arnau’s talk.
The model dependence associated with the use of different many-body approaches is not large in neutron matter. The discrepancy emerging in the case of symmetric nuclear matter is mostly arising from the treatment of the spin-orbit term of the nucleon-nucleon potential.
From the bare NN potential to the effective interaction

Due to the strong short-range repulsion, the nucleon-nucleon potential cannot be used to carry out perturbation theory in the Fermi gas basis.

★ In G-matrix perturbation theory the bare potential is replaced by a well-behaved operator defined through

\[
\langle ij | G(E) | kl \rangle = G_{ij,kl}(E) = v_{ij,kl} + \sum_{mn} v_{ij,mn} \frac{Q_{mn}}{E - \epsilon_m - \epsilon_n + i\eta} G_{mn,kl}(E),
\]

and the ground state energy per baryon is given by

\[
\frac{E}{N_B} = \frac{3}{5} \frac{k_F^2}{2m} + \frac{1}{2} \sum_{i,j \in \{F\}} \langle ij | G(E = \epsilon_i + \epsilon_j) | ij \rangle_a.
\]
In CBF perturbation theory a complete set of correlated states are obtained from the Fermi gas states through the transformation

\[ |n\rangle = F |n_{FG}\rangle = S \prod_{j>i} f_{ij} |n_{FG}\rangle \]
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\( f_{ij} \) determined from functional minimization of

\[
\frac{1}{N_B} \frac{\langle 0 | H | 0 \rangle}{\langle 0 | 0 \rangle} \approx \frac{3}{5} \frac{k_F^2}{2m} + \frac{1}{2} \sum_{i, j \in \{F\}} \langle ij | V_{\text{eff}} | ij \rangle_a
\]

\[ V_{\text{eff}} = f_{12}^\dagger \left[ -\frac{1}{m} (\nabla^2 f_{12}) - \frac{2}{m} (\nabla f_{12}) \cdot \nabla + v_{12} f_{12} \right] \]
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Total neutron-neutron x-section. Argonne $\nu'_6$ potential.
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Total neutron-neutron x-section. Argonne $v'_6$ potential

The effects of the three-nucleon force have also been included in the CBF effective interaction, through a density-dependent two-nucleon interaction (see Alessandro Lovato’s poster).
★ Results obtained using the Argonne $\nu'_6$ potential

★ Medium effects are large. The model dependence is not critical, although it can be clearly seen in the case of viscosity at supranuclear density.
Shear viscosity of $\beta$-stable $npe$ matter

- Required inputs [proton (and electron) fraction, effective masses & scattering rates] obtained from the CBF effective interaction [OB & A. Carbone, PRC, in press]

- Increasing the electron fraction leads to a significant modification of the balance between the different contributions to the viscosity.

- consistency is a critical issue
The contribution of three-nucleon forces can be included in the CBF effective interaction through a density-dependent effective potential. [A. Lovato et al. arXiv:1011.3784, PCR, in press]
Transport coefficients from different many-body approaches

- Brueckner Hartree-Fock calculations including two- and three-nucleon forces (Bonn B parametrization) [H.F. Zhang, U. Lombardo and W. Zuo, Phys. Rev. C 82, 015805 (2010)]
The neutrino mean free path can be obtained from the scattering cross section, expressed in terms of density and spin responses of nuclear matter

\[
\frac{d\sigma}{d\omega d\Omega} = \frac{G_F^2}{8\pi^3} \left[ c_V^2 (1 + \cos \theta)S_0(q, \omega) + C_A^2 (3 - \cos \theta)S_\sigma(q, \omega) \right]
\]

\[
S_x(q, \omega) = \sum_n |\langle n|O_x|0\rangle|^2 \delta(E_0 + \omega - E_n)
\]

\[
\lambda^{-1} = \int d\Omega d\omega \frac{d\sigma}{d\omega d\Omega}
\]

The response functions can be computed within the approach based on the CBF effective interaction.
Nuclear matter response

★ Consider the density response, for simplicity

\[
S(q, \omega) = \sum_n | \sum_k \langle n| a_{k+q}^\dagger a_k |0\rangle|^2 \delta(\omega + E_0 - E_n)
\]

\[
= \int \frac{dt}{2\pi} e^{i(\omega+E_0)t} \sum_{p,k} \langle 0| a_{p+q} a_p^\dagger e^{-iHt} a_{k+q}^\dagger a_k |0\rangle
\]
Calculate of the particle-hole propagator

Define effective operators consistent with the effective interaction (same correlation functions & same cluster order)

\[
(p_h|O_{\text{eff}}|0) = (p_h| \sum_{j > i} f_{ij} O \sum_{j > i} f_{ij}|0) = (p_h|(1 + \sum_{j > i} g_{ij})O(1 + \sum_{j > i} g_{ij})|0)
\]

\[
O_{\text{eff}} = O + N(N - 1) \{O, g_{ij}\} + g_{ij}Og_{ij}
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O_{\text{eff}} = O + N(N - 1) [\{O, g_{ij}\} + g_{ij} O g_{ij}]
\]

\[S(q, \omega)\] can be expressed in terms of \(O_{\text{eff}}\) the ph propagator \(\Pi_{\text{ph}}\)
\( S(q, \omega) \) at low momentum transfer

- Short- and long-range correlations are consistently taken into account by \( O_{\text{eff}} \) and \( V_{\text{eff}} \), respectively. Long-range correlations are dominant at low momentum transfer.

- In the Tamm Dancoff (TD) approximation, the final state is expanded in the basis of 1p1h states \([\text{OB} \& \text{N.Farina PLB 680, 305 (2009)}]\)

\[
|f\rangle = |q, \text{TSM}\rangle = \sum_i c_i^{\text{TSM}} |p_i h_i, \text{TSM}\rangle ,
\]

- At fixed \( q \), the energy of the state \( |f\rangle , \omega_f \), and the coefficients \( c_i^{\text{TSM}} \), are determined diagonalizing the matrix

\[
H_{ij}^{\text{TSM}} = (E_0 + e_{pi} - e_{hi}) \delta_{ij} + (h_{ip_i}, \text{TSM}|V_{\text{eff}}|h_{jp_j}, \text{TSM}) ,
\]

\[
S(q, \omega) = \sum_{\text{TSM}} \sum_n \left| \sum_i (c_n^{\text{TSM}})_i (h_{ip_i}, \text{TSM}|O_{\text{eff}}|0) \right|^2 \delta(\omega - \omega_n^{\text{TSM}}) ,
\]
Symmetric nuclear matter at equilibrium density

- mean field + short-range-correlations $\rightarrow$ shift at larger $\omega$ + quenching of the transition matrix elements
- long-range-correlations $\rightarrow$ excitation of a collective mode at low $|q|$
Emissivity due to bremsstrahlung of $\nu - \bar{\nu}$ pairs

In Born approximation, the emission rate of the process

\begin{align*}
\nu \rightarrow n(p_1) n(p_2) n(p_3) n(p_4) \nu - \nu
\end{align*}

is driven by the trace

\begin{align*}
H^{ii} &= 16 \frac{1}{\omega^2} \sum_{M_SM_{S'}} |\langle 1M_{S'} | [S_i, V_{\text{eff}}(q)] | 1M_S \rangle|^2.
\end{align*}

where $S_i$ denotes the $i$-icomponent of the total spin
One Pion Exchange (OPEP) vs CBF effective interaction

- Nuclear dynamics beyond OPEP: factor $\sim 4 \div 5$ (Reddy et al, AD 2001)
- Screening due to neutron-neutron correlations: factor $\sim 6 \div 7$
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- Example: input needed for the analysis of r-mode instability of rotating matter composition
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- shear and bulk viscosity coefficients
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Open issues include the fully quantitative description of symmetric nuclear matter and a systematic (and meaningful) comparison with dynamical models including relativistic effects.