Gravitational collapse to third family compact stars

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Motivation

Equation of State at Supranuclear Densities and the Existence of a Third Family of Superdense Stars

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This paper presents a method for deducing the equation of state of "cold" matter at supranuclear densities from astronomical data. In particular, from the masses and the radii of a sequence of superdense stars composed of degenerate matter, one can determine the equation of state. The relationship between the equation of state and the mass-radius curve is used to construct an equation of state that allows a third family of superdense stars.

INTRODUCTION

REASONS have been advanced for believing that the birth of a neutron star coincides with the occurrence of a supernova explosion. The mass regime within which neutron stars are calculated to be stable is approximately \( 0.15M_\odot \) to \( 0.7M_\odot \); the exact numbers depend upon an exact knowledge of the equation of state, i.e., the nucleon-nucleon interaction. The central density of the neutron core resulting from a supernova can range—it is calculated—from below nuclear densities, \( \sim 2 \times 10^{15} \text{ g/cm}^3 \), to about 20 times nuclear densities, \( \sim 6 \times 10^{18} \text{ g/cm}^3 \). At these and higher densities several workers have suggested that hyperons appear.

Thus, because of our ignorance of nuclear interactions at superhigh densities, one cannot exclude the possibility that one or another elementary-particle transformation may strongly influence the compressibility of matter at a certain supranuclear density. In that event the stability of a superdense star at these densities may be affected quite significantly. Significantly enough the properties of the two already family of superdense stars? This effect, though less likely, would be far more dramatic and decisive in what it would tell about the equation of state. Therefore, this paper asks and answers this question: How must the equation of state run to permit a third family of degenerate stars?

In answering this query we are led to formulate and answer a more general question:

Given the mass-radius relation \( M = M(R) \) for the entire sequence of stars associated with a given equation of state, \( p = p(\rho) \), find that equation of state.

The converse problem is well-known and thoroughly studied: Given the equation of state, find the family of equilibrium configurations. Starting with a given value of the central density \( \rho = \rho_0 \), one integrates the equations of hydrostatic equilibrium

\[
\frac{dp^*}{dr} = -\frac{(p^* + \rho^*)(m^* + 4\pi \rho^*)}{r^2 - 2m^*r}, \tag{1}
\]

\[
\frac{dm^*}{dr} = 4\pi \rho^* \tag{2}
\]
Motivation

Necessary condition for stability with respect to radial oscillations:

\[
\left( \frac{\partial M}{\partial \epsilon_c} \right)^0_j > 0
\]

\[
\frac{\partial R}{\partial \epsilon_c} < 0 \rightarrow \text{one even mode changes}
\]

\[
\frac{\partial R}{\partial \epsilon_c} > 0 \rightarrow \text{one odd mode changes}
\]

Goal: studying phase transition in the dynamic background of collapsing neutron star

Strong neutrino, GW emission expected during transition between "second family" and "third family"

\[ \Omega_{CFL}^{\text{quarks}} = \frac{6}{\pi^2} \int_0^\nu p^2 (p - \mu) \, dp + \frac{3}{\pi^2} \int_0^\nu p^2 \left( \sqrt{p^2 + m^2_s} - \mu \right) \, dp - \frac{3\Delta^2 \mu^2}{\pi^2} + B \quad \nu \equiv 2\mu - \sqrt{\mu^2 + \frac{m^3_s}{3}} \]

- TM1 EOS for the outer hadronic phase (relativistic mean-field model)
- CFL EOS for the inner deconfined phase

catastrophic rearrangement

assumption: total baryon number and angular momentum $J$ conserved during catastrophic rearrangement

$$J = 2\pi \int d^2 x \ T^0_3 \sqrt{-g}$$

 gained energy during rearrangement: $\Delta M \sim 10^{52} \text{erg}$  enhancement due to rotation
catastrophic rearrangement

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\[ J = 2\pi \int d^2x \ T^0_3 \sqrt{-g} \]

\[
\begin{array}{cccc}
\text{equatorial radius [km]} & 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1 & 1.2 & 1.4 & 1.6 & 1.8 & 2 & 2.2 \\
\text{gravitational mass [solar unit]} & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{TM1 constant frequency 900 Hz} & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\text{130 50 160 0.2 constant angular momentum} & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\text{130 50 165 0.2 constant angular momentum} & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ & \circ \\
\end{array}
\]

gained energy during rearrangement: $\Delta M \sim 10^{52} \text{erg}$ enhancement due to rotation

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Simplified dynamical model (Newtonian ideal hydro)

\[
\frac{\partial N}{\partial t} + \nabla \cdot (N \mathbf{v}) = 0 \quad \text{baryon current conservation}
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot (E \mathbf{v}) + \nabla \cdot (P \mathbf{v}) = 0 \quad \text{energy conservation}
\]

\[
\frac{\partial \mathbf{M}}{\partial t} + (\nabla \cdot \mathbf{M}) \mathbf{v} + (\mathbf{M} \cdot \nabla) \mathbf{v} + \nabla P = 0 \quad \text{momentum conservation}
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1 - \mathbf{v}^2}{\rho + P} \left[ \nabla P + \mathbf{v} \frac{\partial P}{\partial t} \right] = 0 \quad \text{Euler’s equation}
\]

Application to spherical 2-phase-star (constant baryon density each) leads to 2nd order ODE governing the evolution of the shockfront

\[
0 = A + B + C + D
\]

\[
A = (1 - \lambda) \left( 2R_c^2 + \dot{R}_c \right) \left( 1 - \frac{R_c}{\left( R_i^3 - (\lambda - 1) R_c^3 \right)^{\frac{1}{3}}} \right)
\]

\[
B = \frac{1}{2} (\lambda - 1)^2 R_c^4 \dot{R}_c^2 \left( \frac{1}{\left( R_i^3 - (\lambda - 1) R_c^3 \right)^\frac{4}{3}} - \frac{1}{R_c^4} \right)
\]

\[
C = \begin{cases} 
-\frac{2}{3} \pi \rho_b R_i^2 + \lambda (\lambda - 1) \dot{R}_c^2 & \text{if } \dot{R}_c < 0 \\
-\frac{2}{3} \pi \rho_b R_i^2 & \text{if } \dot{R}_c > 0
\end{cases}
\]

\[
D = \frac{4}{3} \pi (\rho_b - \rho_a) R_c^2 \left( \frac{R_c}{\left( R_i^3 - (\lambda - 1) R_c^3 \right)^{\frac{1}{3}}} - 1 \right) + \frac{2}{3} \pi \rho_b \left( R_s^2 - R_c^2 \right)
\]


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\[ \frac{\partial N}{\partial t} + \nabla \cdot (N \mathbf{v}) = 0 \]

baryon current conservation

\[ \frac{\partial E}{\partial t} + \nabla \cdot (E \mathbf{v}) + \nabla \cdot (P \mathbf{v}) = 0 \]

energy conservation

\[ \frac{\partial M}{\partial t} + (\nabla \cdot M) \mathbf{v} + (M \cdot \nabla) \mathbf{v} + \nabla P = 0 \]

momentum conservation

\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{\rho + P} \left[ \nabla P + \mathbf{v} \frac{\partial P}{\partial t} \right] = 0 \]

Euler’s equation

Application to spherical 2-phase-star (constant baryon density each) leads to 2nd order ODE governing the evolution of the shockfront

Schwarzschild-like metric for spherical mass-distribution with radial motion allowed

$$g_{\mu\nu} = \begin{pmatrix} -e^{2\Phi(r,t)} & 0 & 0 & 0 \\ 0 & e^{2\Lambda(r,t)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix}$$

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2}g^{\lambda\rho} [\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}]$$

17 nonvanishing connection coefficients

$$\begin{align*}
\Gamma^0_{00} &= \dot{\Phi} \\
\Gamma^0_{01} &= \Phi' \\
\Gamma^0_{11} &= e^{2\Lambda-2\Phi} \dot{\Lambda} \\
\Gamma^1_{00} &= e^{2\Phi-2\Lambda} \Phi' \\
\Gamma^1_{01} &= \dot{\Lambda} \\
\Gamma^1_{11} &= \Lambda' \\
\Gamma^1_{22} &= -r e^{-2\Lambda} \\
\Gamma^1_{33} &= -r \sin^2 \theta e^{-2\Lambda} \\
\Gamma^2_{12} &= r^{-1} \\
\Gamma^2_{33} &= -\sin \theta \cos \theta \\
\Gamma^3_{13} &= r^{-1} \\
\Gamma^3_{23} &= \frac{\cos \theta}{\sin \theta} \\
\Gamma^0_{0\rho} &= \dot{\Phi} + \dot{\Lambda} \\
\Gamma^0_{1\rho} &= \Phi' + \Lambda' + 2r^{-1}
\end{align*}$$

-Tolman, Relativity Thermodynamics and Cosmology, 1934, page 250 ff.

\[ R_{\mu \nu} = \partial_\rho \Gamma^{\rho}_{\mu \nu} - \partial_\nu \Gamma^{\rho}_{\mu \rho} + \Gamma^{\rho}_{\lambda \rho} \Gamma^{\lambda}_{\mu \nu} - \Gamma^{\rho}_{\nu \lambda} \Gamma^{\lambda}_{\mu \rho} \]

\( R_{01} \) only nondiagonal nonvanishing component due to spherycally symmetric pulsation

\[ R_{00} = e^{2\Phi - 2\Lambda} \left[ -\Phi' \Lambda' + \Phi'' + \Phi'^2 + 2r^{-1} \Phi' \right] + \Phi \dot{\Lambda} - \dot{\Lambda}^2 - \dot{\Lambda} \]

\[ R_{01} = 2r^{-1} \dot{\Lambda} \]

\[ R_{11} = e^{2\Lambda - 2\Phi} \left[ -\Phi \dot{\Lambda} + \dot{\Lambda} + \dot{\Lambda}^2 \right] - \Phi'' + \Phi' \Lambda' + 2r^{-1} \Lambda' - \Phi'^2 \]

\[ R_{22} = -e^{-2\Lambda} + r\Lambda' e^{-2\Lambda} - r\Phi' e^{-2\Lambda} + 1 \]

\[ R_{33} = \sin^2 \theta \left[ -e^{-2\Lambda} + r\Lambda' e^{-2\Lambda} - r\Phi' e^{-2\Lambda} + 1 \right] \]

\[ R = R^\rho_\rho = -2e^{-2\Lambda} \left[ \Phi'' - \Phi' \Lambda' + \Phi'^2 - 2r^{-1} \left( \Lambda' - \Phi' \right) + \left( 1 - e^{2\Lambda} \right) r^{-2} \right] + 2e^{-2\Phi} \left[ -\Phi \dot{\Lambda} + \dot{\Lambda} + \dot{\Lambda}^2 \right] \]
Einstein tensor, Energy-momentum tensor

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \]

\( G_{00} = e^{2\Phi - 2\Lambda} \left[ 2r^{-1}\Lambda' - \left( 1 - e^{2\Lambda} \right) r^{-2} \right] \) time-derivatives cancel out

\( G_{01} = 2r^{-1}\dot{\Lambda} \)

\( G_{11} = 2r^{-1}\Phi' + \left( 1 - e^{2\Lambda} \right) r^{-2} \) time-derivatives cancel out

\( G_{22} = e^{-2\Lambda} \left[ r^2 \Phi''' - r^2 \Phi' \Lambda' + r^2 \Phi'^2 + r \left( \Phi' - \Lambda' \right) \right] + e^{-2\Phi} \left[ r^2 \Phi \dot{\Lambda} - r^2 \dot{\Lambda} - r^2 \dot{\Lambda}'^2 \right] \)

\( G_{33} = \sin^2 \theta e^{-2\Lambda} \left[ r^2 \Phi''' - r^2 \Phi' \Lambda' + r^2 \Phi'^2 + r \left( \Phi' - \Lambda' \right) \right] + \sin^2 \theta e^{-2\Phi} \left[ r^2 \Phi \dot{\Lambda} - r^2 \dot{\Lambda} - r^2 \dot{\Lambda}'^2 \right] \)

Energy-momentum tensor for an ideal electrically charged liquid

\[ T_{\mu\nu} = (\rho + P) u_\mu u_\nu + P g_{\mu\nu} + \frac{1}{4\pi} \left[ F_\mu^\alpha F_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} F^\zeta_\xi F_\zeta_\xi \right] \]

assuming spherically symmetric motion: \( u_0 > 0, \ |u_1| < 1, \ u_2 = 0, \ u_3 = 0 \)
normalisation of 4-velocity

\[ 1 = -u^\mu u_\mu = e^{2\Phi} u^0 - e^{2\Lambda} u^1 = u^0 [e^{2\Phi} - e^{2\Lambda} v^2] \]

\[ \rightarrow u^0 = (e^{2\Phi} - e^{2\Lambda} v^2)^{-\frac{1}{2}} \quad u^1 = v (e^{2\Phi} - e^{2\Lambda} v^2)^{-\frac{1}{2}} \]

Due to spherical symmetry only \( F^{01} = E, F^{10} = -E \) is nonvanishing

\[ \partial_1 \left[ \sqrt{-g} F^{01} \right] = 4\pi \sqrt{-g} j^0 \rightarrow Q(r) \equiv 4\pi \int_0^r r' e^{\Phi + \Lambda} j^0 dr' = e^{\Phi + \Lambda} r E \]

\[ T_{00} = \frac{\rho e^{4\Phi} + P e^{2\Phi + 2\Lambda} v^2}{e^{2\Phi} - e^{2\Lambda} v^2} + \frac{Q^2 e^{2\Phi}}{8\pi r^4} \]

\[ T_{01} = -(\rho + P) \frac{e^{2\Phi + 2\Lambda} v}{e^{2\Phi} - e^{2\Lambda} v^2} \]

\[ T_{11} = \frac{P e^{2\Phi + 2\Lambda} + \rho e^{4\Lambda} v^2}{e^{2\Phi} - e^{2\Lambda} v^2} - \frac{Q^2 e^{2\Lambda}}{8\pi r^4} \]

\[ T_{22} = Pr^2 + r^2 \frac{Q^2}{8\pi r^4} \]

\[ T_{33} = Pr^2 \sin^2 \theta + r^2 \sin^2 \theta \frac{Q^2}{8\pi r^4} \]
Fluid equations I

"structure and evolution" equations are derived from

- baryon number conservation \( n_B u^\mu;_\mu = 0 \)
- energy-momentum conservation \( T^{\mu\nu;}_\nu = 0 \)
- Einstein equation \( G_{\mu\nu} - 8\pi T_{\mu\nu} = 0 \)
- Maxwell equation \( \partial_\mu \left[ \sqrt{-g} F^{\nu\mu} \right] - 4\pi \sqrt{-g} j^\nu = 0 \)

assuming an isentropic star (one-parameter equation of state)

\[
e^{-2\Lambda} = \frac{r^2 - 2mr + Q^2}{r^2}
\]

\[
\partial_\mu \left[ \sqrt{-g} F^{0\mu} \right] - 4\pi \sqrt{-g} j^0 = 0 \quad \Rightarrow \quad \frac{\partial Q}{\partial r} = \frac{4\pi \rho_{ch} r^2 e^{\Phi + \Lambda}}{(e^{2\Phi} - e^{2\Lambda} v^2)^{1/2}}
\]

\[
\partial_\mu \left[ \sqrt{-g} F^{1\mu} \right] - 4\pi \sqrt{-g} j^1 = 0 \quad \Rightarrow \quad \frac{\partial Q}{\partial t} = -\frac{4\pi \rho_{ch} r^2 e^{\Phi + \Lambda} v}{(e^{2\Phi} - e^{2\Lambda} v^2)^{1/2}}
\]

advection equation for charge: \( \dot{Q} + vQ' = 0 \)
Fluid equations II

\[ G_{00} - 8\pi T_{00} = 0 \rightarrow \frac{\partial m}{\partial r} = 4\pi r^2 \frac{\rho e^{2\Phi} + Pe^{2\Lambda}v^2}{e^{2\Phi} - e^{2\Lambda}v^2} + \frac{Q Q'}{r} \]

\[ G_{01} - 8\pi T_{01} = 0 \rightarrow \frac{\partial m}{\partial t} = -\frac{4\pi r^2 (\rho + P)e^{2\Phi}v}{e^{2\Phi} - e^{2\Lambda}v^2} + \frac{Q \dot{Q}}{r} \]

\[ G_{11} - 8\pi T_{11} = 0 \rightarrow \frac{\partial \Phi}{\partial r} = \frac{mr - Q^2 + 4\pi r^4 \frac{Pe^{2\Phi} + \rho e^{2\Lambda}v^2}{e^{2\Phi} - e^{2\Lambda}v^2}}{r^3 - 2mr^2 + Q^2 r} \]

\[ T^\mu_\nu = 0 \rightarrow \frac{\partial P}{\partial r} = \sqrt{e^{2\Phi - 2\Lambda - v^2}} \left\{ \frac{Q \rho c h e^{\Phi} - e^{2\Phi} (\rho + P)}{e^{2\Phi} (r^2 - 2mr^2 + Q^2)} \right\} - \frac{Q \rho c h e^{\Phi} v}{e^{2\Phi} (r^2 - 2mr^2 + Q^2)} \]

\[ T^\mu_\nu = 0 \rightarrow \frac{\partial P}{\partial t} = \frac{e^{2\Phi} (\rho + P) \left[ 6mv - 2rv + 8\pi Pr^3 + 2mr v' - r^2 v' - Q^2 \left( 4r^{-1} v + v' \right) \right]}{e^{2\Phi} (r^2 - 2mr^2 + Q^2)} \]

\[ (n_B u^\mu)_;_\mu = 0 \rightarrow \frac{\partial v}{\partial t} = \text{long expression} \]

advection equation for pressure: \[ \dot{P} + vP' = 0 \quad \text{for } r > 0 \]

time evolution of central pressure: \[ \dot{P} = - (\rho + P) v' \quad \text{for } r = 0 \]

open question: analytic form of \[ \frac{\partial \Phi}{\partial t} \]
Fluid equations II - chargeless limit

\[
G_{00} - 8\pi T_{00} = 0 \Rightarrow \frac{\partial m}{\partial r} = 4\pi r^2 \frac{\rho e^{2\Phi} + Pe^{2\Lambda}v^2}{e^{2\Phi} - e^{2\Lambda}v^2}
\]

\[
G_{01} - 8\pi T_{01} = 0 \Rightarrow \frac{\partial m}{\partial t} = -\frac{4\pi r^2 (\rho + P) e^{2\Phi}v}{e^{2\Phi} - e^{2\Lambda}v^2}
\]

\[
G_{11} - 8\pi T_{11} = 0 \Rightarrow \frac{\partial \Phi}{\partial r} = \frac{m + 4\pi r^3 \rho e^{2\Phi} + Pe^{2\Lambda}v^2}{r (r - 2m)}
\]

\[
T^\mu_{\nu;\mu} = 0 \Rightarrow \frac{\partial P}{\partial r} = -\frac{e^{2\Phi} (\rho + P) \left[6m - 2r + 8\pi Pr^3 + 2mr^{-1}v' - r^2v^{-1}v'\right]}{e^{2\Phi} (r^2 - 2mr) - r^2v^2}
\]

\[
T^\mu_{\nu;\mu} = 0 \Rightarrow \frac{\partial P}{\partial t} = \frac{e^{2\Phi} (\rho + P) \left[6mv - 2rv + 8\pi Pr^3v + 2mrv' - r^2v'\right]}{e^{2\Phi} (r^2 - 2mr) - r^2v^2}
\]

\[
(n_B u^\mu)_{;\mu} = 0 \Rightarrow \frac{\partial v}{\partial t} = \text{long expression}
\]

advection equation for pressure: \(\dot{P} + vP' = 0\) for \(r > 0\)

time evolution of central pressure: \(\dot{P} = -(\rho + P)v'\) for \(r = 0\)

open question: analytic form of \(\frac{\partial \Phi}{\partial t}\)
Fluid equations II - static limit

\[ G_{00} - 8\pi T_{00} = 0 \rightarrow \frac{\partial m}{\partial r} = 4\pi \rho r^2 + \frac{Q}{r} \]

\[ G_{01} - 8\pi T_{01} = 0 \rightarrow \frac{\partial m}{\partial t} = 0 \]

\[ G_{11} - 8\pi T_{11} = 0 \rightarrow \frac{\partial \Phi}{\partial r} = \frac{mr - Q^2 + 4\pi Pr^4}{r^3 - 2mr^2 + Q^2 r} \]

\[ T^{\mu \nu}_{;\mu} = 0 \rightarrow \frac{\partial P}{\partial r} = -\frac{(\rho + P) \left[ mr - Q^2 + 4\pi Pr^4 \right]}{r^3 - 2mr^2 + Q^2 r} + \frac{Q}{4\pi r^4} \frac{Q'}{r} \]

\[ T^{\mu \nu}_{;\mu} = 0 \rightarrow \frac{\partial P}{\partial t} = 0 \]

\[ (n_B u^\mu)_{;\mu} = 0 \rightarrow \frac{\partial v}{\partial t} = 0 \]

Jacob David Bekenstein, Phys. Rev. D 4, 21852190 (1971)
Fluid equations II - static & chargeless limit

\[ G_{00} - 8\pi T_{00} = 0 \rightarrow \frac{\partial m}{\partial r} = 4\pi \rho r^2 \]

\[ G_{01} - 8\pi T_{01} = 0 \rightarrow \frac{\partial m}{\partial t} = 0 \]

\[ G_{11} - 8\pi T_{11} = 0 \rightarrow \frac{\partial \Phi}{\partial r} = \frac{m + 4\pi Pr^3}{r^2 - 2mr} \]

\[ T^{\mu\nu}_{;\mu} = 0 \rightarrow \frac{\partial P}{\partial r} = -\frac{(\rho + P) \left[ m + 4\pi Pr^3 \right]}{r^2 - 2mr} \]

\[ T^{\mu\nu}_{;\mu} = 0 \rightarrow \frac{\partial P}{\partial t} = 0 \]

\[ (n_B u^\mu)_{;\mu} = 0 \rightarrow \frac{\partial v}{\partial t} = 0 \]

Julius Robert Oppenheimer, George Michael Volkoff, Phys. Rev. 55, 374381 (1939)
Future work

- Inclusion of first and second viscosity in simplified hydrodynamical approach
- Temperature evolution due to energy conservation through shockfront $\rightarrow$ neutrino emissivity
- Solving "TOV-generalizing" GR fluid equations $\rightarrow$ comparison with Lagrangian approach and GR1D (caltech) code
- Relativistic Navier-Stokes for spherical systems
- Dynamic stability properties of relativistic charged fluid spheres
- 2D hydrodynamics restricting only $u^\theta = 0$